Chapter 2 Part 1
Motion in one dimension

Sections 2-, 2, 3, 4, 5

2-2 Motion in 1 dimension

We live in a 3-dimensional world, so why bother analyzing 1-dimensional situations? Basically, because any translational (straight-line, as opposed to rotational) motion problem can be separated into one or more 1-dimensional problems. Problems are often analyzed this way in physics; a complex problem can often be reduced to a series of simpler problems.

The first step in solving a problem is to set up a coordinate system. This defines an origin (a starting point) as well as positive and negative directions. We'll also need to distinguish between scalars and vectors. A scalar is something that has only a magnitude, like area or temperature, while a vector has both a magnitude and a direction, like displacement or velocity.

In analyzing the motion of objects, there are four basic parameters to keep track of. These are time, displacement, velocity, and acceleration. Time is a scalar, while the other three are vectors. In 1 dimension, however, it's difficult to see the difference between a scalar and a vector! The difference will be more obvious in 2 dimensions.

2-3 Displacement

The **displacement,** $\Delta\vec{x}$**,** is the **distance moved in a given direction**.

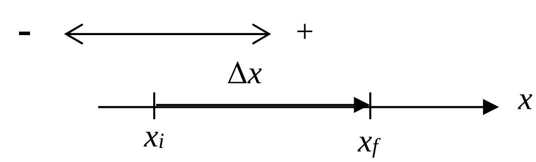
Displacement:

1- is a vector quantity.
2- is positive if the particle moves in the positive direction, and negative if the particle has moved in the negative direction and
3- is measured in meters.

Mathematically, in one dimension, the displacement is defined as:

$$\Delta x = x_f - x_i$$

where, x_f is the final position and x_i is the initial position of the particle. Δx is a vector pointing from x_i to x_f .



Example: if $x_i = 5$ m and $x_f = 12$ m , then

$\Delta x = 12 \text{ m} - 5 \text{ m} = +7 \text{ m}$

Comment: the positive result indicates that the motion is in the positive direction.

Example: if $x_i = 5$ m and $x_f = 1$ m , then $\Delta x = 1 \text{ m} - 5 \text{ m} = -4 \text{ m}$

Comment: the negative result indicates that the motion is in the negative direction.

H.W. Do Checkpoint 1.

4-4 Speed and velocity

Example: Imagine that on your way to class one morning, you leave home on time, and you walk at 3 m/s east towards campus. After exactly one minute you realize that you've left your physics assignment at home, so you turn around and run, at 6 m/s, back to get it. You're running twice as fast as you walked, so it takes half as long (30 seconds) to get home again.

Answer: There are several ways to analyze those 90 seconds between the time you left home and the time you arrived back again. One number to calculate is your average speed, which is defined as the total distance covered divided by the time. If you walked for 60 seconds at 3 m/s, you covered 180 m. You covered the same distance on the way back, so you went 360 m in 90 seconds.

$$\text{total distance} = 3\ \frac{\text{m}}{\text{s}} \times 60\ \text{s}\ + 6\ \frac{\text{m}}{\text{s}} \times 30\ \text{s} = 360\ \text{m},$$

$$\text{displacement} = 3\ \frac{\text{m}}{\text{s}} \times 60\ \text{s}\ - 6\ \frac{\text{m}}{\text{s}} \times 30\ \text{s} = 0\ \text{m},$$

$$\text{Total elapsed time } = 60\ \text{s} + 30\ \text{s} = 90\ \text{s}$$

$$\text{Average speed } s_{\text{avg}} = \frac{\text{total distance traveled}}{\text{total time taken}} = \frac{360}{90} = 4\ \frac{\text{m}}{\text{s}},$$

$$\text{Average velocity } \text{v}_{\text{avg}} = \frac{\text{displacement}}{\text{total time taken}} = \frac{0}{90} = 0\ \frac{\text{m}}{\text{s}},$$

In this case, your average velocity for the round trip is zero, because you're back where you started so the displacement is zero

2-5 Instantaneous velocity and speed

We usually think about speed and velocity in terms of their instantaneous values, which tell us how fast, and, for velocity, in what direction an object is traveling at a particular instant. **The instantaneous velocity is defined as the rate of change of position with time, for a very small time interval**. In a particular time interval Δt , if the displacement is Δx , the velocity during that time interval is:

$$\text{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous speed is simply the magnitude of the instantaneous velocity.

Ex. An electron moving along the x axis has a position given by $x = (16te^{-t})\,\text{m}$, where t is in seconds. How far is the electron from the origin when it momentarily stops?

To find the velocity of the electron as a function of time, take the first derivative of $x(t)$:

$$v = \frac{dx}{dt} = 16e^{-t} - 16te^{-t} = 16e^{-t}(1-t)\,\frac{\text{m}}{\text{s}}$$

again where t is in seconds, so that the units for v are $\frac{\text{m}}{\text{s}}$.

Now the electron "momentarily stops" when the velocity v is zero. From our expression for v we see that this occurs at $t = 1\,\text{s}$. At this particular time we can find the value of x:

$$x(1\,\text{s}) = 16(1)e^{-1}\,\text{m} = 5.89\,\text{m}$$

The electron was 5.89 m from the origin when the velocity was zero.

Acceleration "a": it is the rate of change of velocity. An object accelerates whenever its velocity changes.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Ex. (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when if ever is the particle's velocity zero? (b) When is its acceleration a zero? (c) When is a negative? Positive? (d) Graph $x(t)$, $v(t)$, and $a(t)$.

Answer:

(a) From Eq. 2.3 we find $v(t)$ from $x(t)$:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(20t - 5t^3) = 20 - 15t^2$$

where, if t is in seconds then v will be in $\frac{m}{s}$. The velocity v will be zero when

$$20 - 15t^2 = 0$$

which we can solve for t:

$$15t^2 = 20 \qquad \Longrightarrow \qquad t^2 = \frac{20}{15} = 1.33\,s^2$$

(The units s^2 were inserted since we know t^2 must have these units.) This gives:

$$t = \pm 1.15\,s$$

(We should be careful... t may be meaningful for negative values!)

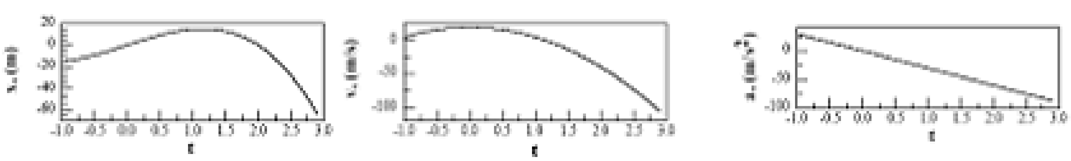
(b) From Eq. 2.5 we find $a(t)$ from $v(t)$:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(20 - 15t^2) = -30t$$

where we mean that if t is given in seconds, a is given in $\frac{m}{s^2}$. From this, we see that a be zero only at $t = 0$.

(c) From the result is part (b) we can also see that a is negative whenever t is positive. positive whenever t is negative (again, assuming that $t < 0$ has meaning for the moti this particle).

(d) Plots of x(t), v(t) and a(t) are given in the following figure.



<u>Average Speed</u>: Usually, we are interested to measure the average speed in two cases:

a- For the whole journey, where

$$s_{avg} = \frac{\text{total distance traveled}}{\text{total time taken}}$$

b- For one part of a trip, where v_i is the initial speed, v_f is the final speed reached in the given part, thus

$$s_{avg} = \frac{1}{2}(v_i + v_f)$$

Instantaneous velocity

We usually think about speed and velocity in terms of their instantaneous values, which tell us how fast, and, for velocity, in what direction an object is traveling at a particular instant. **The instantaneous velocity is defined as the rate of change of position with time, for a very small time interval**. In a particular time interval Δt, if the displacement is Δx, the velocity during that time interval is:

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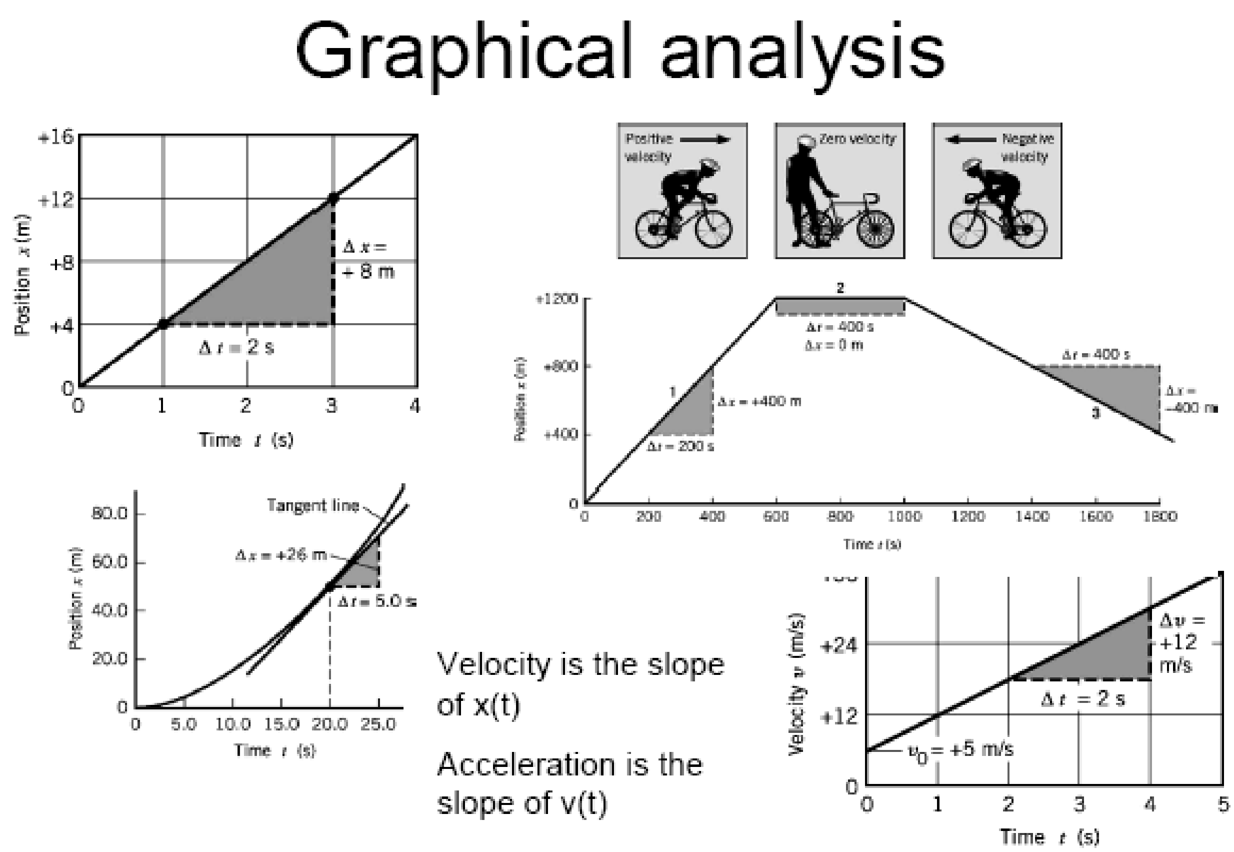
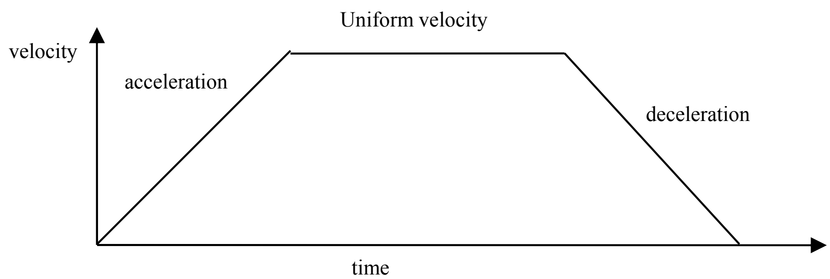


Example: see text sample problem 2.3

<u>Velocity-Time Graphs</u>: The velocity-time graph usually describes a certain trip.



A journey usually starts from rest. The body accelerates and the velocity increases to a certain value, then it travels at a constant speed (uniform velocity), and finally it slows down (decelerates) until it comes to a complete stop at a certain destination.

(*) the slope of the velocity-time graph at a certain moment gives the value of the acceleration at that moment. Notice that:

1- The slope is positive when the body is accelerating (the velocity is increasing).
2- The slope is zero when the velocity is uniform (the velocity remains constant).
3- The slope is negative when the body is decelerating (the velocity is decreasing).

The distance traveled during a trip can be calculated by:

Distance traveled = average speed ×total time

Or

Distance traveled equals the area under the (v,t) graph.

Notes that: $\frac{dx}{dt} = v \Rightarrow x = \int v dt =$ area under the curve

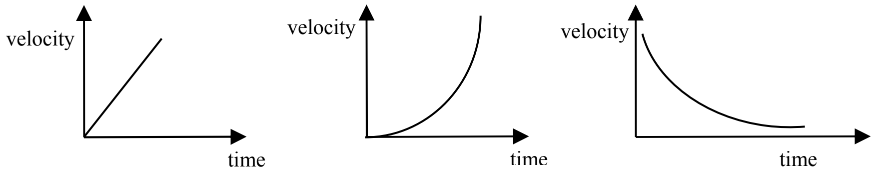
Sections 2-6

Acceleration: "a" is the rate of change of velocity.

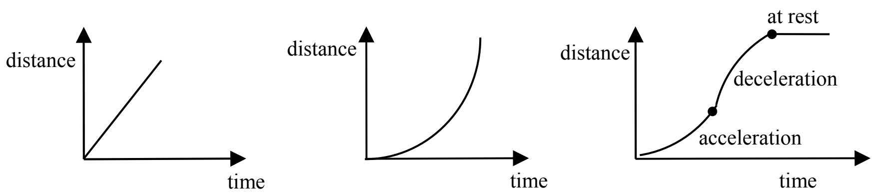
$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{total time}},$$

$$a = \frac{v_f - v_i}{t}$$

i- Acceleration is a vector quantity measured in (m/s²).
ii- Acceleration occurs only during the application of unbalanced force.
iii- Acceleration may remain constant, called "uniform acceleration", or it may increase by time "acceleration", or decrease "deceleration". This can be observed from the shapes of v-t graphs shown:

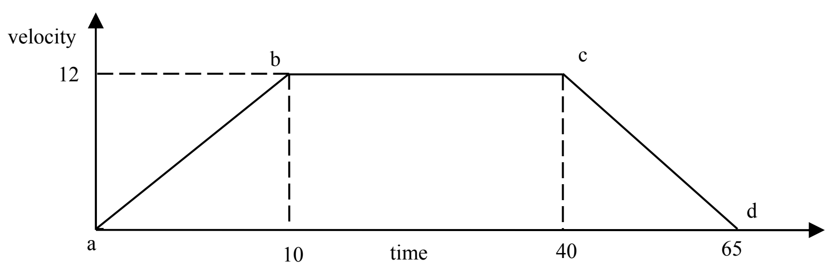


Constant acceleration | Non-uniform acceleration (increasing acceleration) | Non-uniform acceleration (decreasing acceleration)



Constant velocity | Non-uniform velocity (acceleration)

Example: In the velocity-time graph given, find: (a) acceleration, (b) the deceleration, (c) the total distance traveled, (d) the average velocity during the trip.



Answer:

$$\text{Acceleration (ab)} = \frac{v_b - v_a}{t} = \frac{12-0}{10} = 1.2 \ \frac{\text{m}}{\text{s}^2}$$

$$\text{Decelaration (cd)} = \frac{v_d \text{ -} v_c}{t} = \frac{0-12}{25} = -0.48 \ \frac{\text{m}}{\text{s}^2}$$

$$\text{Distance traveled } = \text{area under the trapezium} = \tfrac{1}{2}(65+30)\times 12 = 570 \text{ m}$$

$$\text{Average velocity } = \frac{\text{total distance}}{\text{total time}} = \frac{570}{65} = 8.8 \ \frac{\text{m}}{\text{s}}$$

Instantaneous acceleration "a": it is the rate of change of velocity. An object accelerates whenever its velocity changes**.**

$$\text{a} = \lim_{\Delta t \to 0} \frac{\Delta \text{v}}{\Delta t} = \frac{d\text{v}}{dt}$$

Example: see text sample problem 2.4

Equations of Motion: For motion under constant (uniform) acceleration, most of the problems can be solved by the equations:

$$v_f = v_i + at, \qquad d = \Delta x = \left(x_f - x_i\right) = \tfrac{1}{2}\,(v_i + v_f)t$$

where v_i is the initial velocity, v_f is the final velocity, d is the distance traveled, a is the acceleration, and t is the time taken.

Section 2.7
Kinematics equations when the acceleration is constant

When the acceleration of an object is constant, calculations of the distance traveled by an object, the velocity it's traveling at a particular time, and/or the time it takes to reach a particular velocity or go a particular distance, are simplified. There are four equations that can be used to relate the different variables, so that knowing some of the variables allows the others to be determined.

Note that the equations apply under these conditions:

1. the acceleration is constant
2. the motion is measured from $t = 0$
3. the equations are vector equations, but the variables are not normally written in bold letters. The fact that they are vectors comes in, however, with positive and negative signs.

The required equations are:

0 – $d = x_f - x_i = vt$ (to be used if v is constant, i.e. $a = 0$)

1 – $v_f = v_i + at,$ (to be used if the Δx is not known)

2 – $d = x_f - x_i = v_i t + \frac{1}{2}\, at^2,$ (to be used if the v_f is not known)

3 – $d = x_f - x_i = \frac{1}{2}(v_i + v_f)t,$ (to be used if the a is not known)

4 – $v_f^2 = v_i^2 + 2a\Delta x,$ (to be used if the t is not known)

where v is the instantaneous velocity, v_i is the initial velocity, v_f is the final velocity, x is the position, x_i is the initial position, x_f is the final position, and a is the acceleration.

Example: An object traveling at 8 m/s accelerates at 2 m/s² for a period of 10 seconds.

a- What is the final velocity?

b- How far does it travel during this period?

Answer:

Inputs: $v_i = 8\frac{\text{m}}{\text{s}}; \quad a = 2\frac{\text{m}}{\text{s}^2}; \quad t = 10\text{ s}$

$a-$ $$v_f = v_i + at = 8 + 2\times 10 = \underline{28\,\frac{\text{m}}{\text{s}}}$$

$b-$ $$s = \text{v}_{\text{avg}}\,t = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(8+28)\times 10 = \underline{180\text{ m}}$$

For part b, we can also use the relation:

$$d = x_f - x_i = v_i t + \frac{1}{2}at^2 = 8\times 10 + \frac{1}{2}\times 2\times 10^2 = \underline{180\text{ m}}$$

Example: A body, moving with uniform acceleration, has a velocity of $v_i = 0.12\frac{\text{m}}{\text{s}}$ when its x coordinate is 0.03 m. If its x coordinate 2.00 s later is −0.05 m, what is the magnitude of its acceleration?

Answer:

Inputs: $x_i = 0.03\text{ m}; \; v_i = 0.12\frac{\text{m}}{\text{s}}; \quad x_f = -0.03\text{ m}; \quad t = 2\text{ s}$

Using the equation:

$$d = x_f - x_i = v_i t + \frac{1}{2}at^2 \Rightarrow \frac{1}{2}at^2 = x_f - x_i - v_i t$$

Substitute inputs:

$$\frac{1}{2}at^2 = x_f - x_i - v_i t = -0.05\text{ m} - 0.03\text{ m} - \left(0.12\frac{\text{m}}{\text{s}}\right)\times(2\text{ s})$$

$$= -0.32\text{ m}$$

Solve for a:

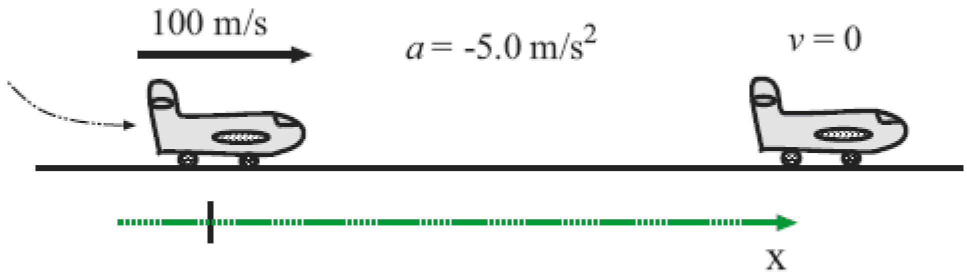
$$a = \frac{2(-0.32\text{ m})}{t^2} = \frac{2(-0.32\text{ m})}{(2\text{ s})^2} = -(0.16\text{ m/s}^2)$$

The x acceleration of the object is (-0.16 m/s^2). (The magnitude of the acceleration is (0.16 m/s^2))

Example: A jet plane lands with a velocity of 100 m/s and can accelerate at a maximum rate of $a = -5.0\text{ m/s}^2$ as it comes to rest.

a- From the instant it touches the runway, what is the minimum time needed before it stops?

b- Can this plane land at a small airport where the runway is 0.80 km long?



Plane touches down on runway at 100 m/s and comes to stop.

Answer:

a- The data given in the problem is illustrated in the above Figure. The minus sign in the acceleration indicates that the sense of the acceleration is opposite that of the motion, that is, the plane is decelerating. The plane will stop as quickly as possible if the acceleration does have the value $a = -5.0 \text{ m/s}^2$, so we use this value in finding the time t in which the velocity changes from $v_i = 100 \text{ m/s}$ to $v_f = 0$. Use the equation:

$$t = \frac{v_f - v_i}{a}$$

Substituting, we find:

$$t = \frac{0 - 100 \frac{\text{m}}{\text{s}}}{-5.0 \frac{\text{m}}{\text{s}^2}} = \underline{20 \text{ s}}$$

The plane needs 20 s to come to stop.

b- The plane also travels the shortest distance in stopping if its acceleration is $a = -5.0 \text{ m/s}^2$. With $x_i = 0$, we can find the plane's final x coordinate using the equation,

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

Using $t = 20$ s which we got from part (a):

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 0 + \left(100 \frac{\text{m}}{\text{s}}\right) \times (20 \text{ s}) + \frac{1}{2}\left(-5.0 \frac{\text{m}}{\text{s}^2}\right)(20 \text{ s})^2$$

$$= \underline{1000 \text{ m}}$$

Or, we can use:

$$x_f = x_i + \frac{1}{2}(v_i + v_f)t = 0 + \frac{1}{2}\left(100 \frac{\text{m}}{\text{s}} + 0\right) \times (20 \text{ s})$$

$$= \underline{1000 \text{ m}}$$

So, the plane must have at least 1.0 km of runway in order to come to rest safely. 0.80 km is not sufficient.

Extra Problems (Chapter 2_I):

Q: A car averages 55.0 mph for the first 4.0 hours of a trip and averages 70.0 mph for each additional hour. The average speed for the entire trip was 60.0 mph. How long is the trip?

Answer:

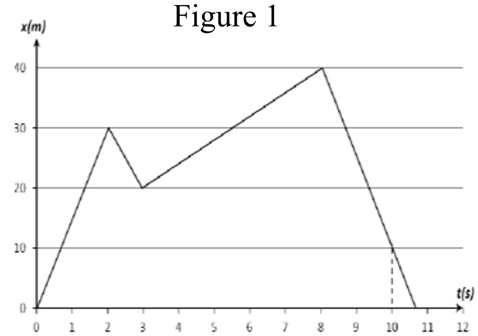
$$v_{ave} = \frac{\Delta x}{\Delta t} = \frac{v_1 t_1 + v_2 t_2}{\Delta t} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{55.0 \times 4.0 + 70.0 \times t_2}{4.0 + t_2} = 60.0$$

$$220.0 + 70.0 \times t_2 = 240.0 + 60.0 t_2 \Rightarrow 10.0 t_2 = 20 \Rightarrow t_2 = 2.0 \text{ hours}$$

$$\therefore \text{ Total Time} = t_1 + t_2 = 4.0 + 2.0 = 6.0 \text{ hours}$$

Q: A particle is moving back and forth along the x-axis. A graph of its position versus time is given in Figure 1. What is the average speed of the particle between t = 0 and t = 10 s?

Figure 1



A) 9 m/s
B) 8 m/s
C) 7 m/s
D) 6 m/s
E) 5 m/s

Answer:

$$v = \left(\frac{30 + 10 + 20 + 30}{10}\right) \text{m/s} = 9 \text{ m/s}$$

What is the average velocity of the particle between t = 0 and t = 10 s?

Ans: 1 m/s

Q: A rocket accelerates vertically up from ground level from rest at 30 m/s^2 for 30 s; then runs out of fuel. What is the rocket's final altitude? (Ignore air resistance)

A) 55 km
B) 45 km
C) 35 km
D) 25 km
E) 15 km

Answer:

$$h_1 = \frac{1}{2} a \times t_1^2, \quad v_1 = a \times t_1, \quad v_1^2 = 2g \times h_2 \rightarrow h_2 = \frac{v_1^2}{2g}$$

$$h = h_1 + h_2 = \frac{1}{2} \times 30 \times 30^2 + \frac{(30 \times 30)^2}{2 \times 9.8} = 55 \text{ km}$$

Q: The position of a particle moving along the x-axis is given by $x(t) = 3t^3 - 9t^2 + 18$, where x is in meters and t is in seconds. What is the value of x when the particle's acceleration is zero?

A) 12 m
B) 10 m
C) 19 m
D) 20 m
E) 15 m

Answer:

$$\frac{dx}{dt} = 9t^2 - 18t$$
$$\frac{d^2x}{dt^2} = 18t - 18 = 0 \text{ at } t = 1\ s$$
$$x(t = 1s) = (3 - 9 + 18)m = 12\ m$$

Q: A time $t = 0$ an object is fired vertically up with an initial speed v_o. It takes the object 10 s to return back to its starting point. What is its initial speed v_o ? (Ignore air resistance)

A) 49 m/s
B) 98 m/s
C) 25 m/s
D) 10 m/s
E) 55 m/s

Answer:

$$v(t) = v_0 - gt$$
$$\text{At } t = 10\ s, -v_0 = v_0 - gt \rightarrow v_0 = \frac{gt}{2} = \frac{9.8 \times 10}{2} = 49\ m/s$$

Q: A car travels up a hill at a constant speed of 40 km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the round trip.

Answer:

Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$v_{ave} = \frac{D_{up} + D_{down}}{t_{up} + t_{down}} = \frac{2D}{\frac{D}{v_{up}} + \frac{D}{v_{down}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

Q: Compute your average velocity in the following two cases:

(a) You walk 73.2 m at a speed of 1.22 m/s and then run 13.2 m at a speed of 3.05 m/s along a straight track.
(b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at3.05 m/s along a straight track.

Answer:

(a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance x = *vt* while the velocity *v* is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

Chapter 2, Part2
Motion in one Dimension

Kinematics equations when the acceleration is constant

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2 − $d = x_f - x_i = v_i t + \frac{1}{2}\, at^2,$ (to be used if the v_f is not known)

3 − $d = x_f - x_i = \frac{1}{2}(v_i + v_f)t,$ (to be used if the a is not known)

4 − $v_f^2 = v_i^2 + 2a\Delta x,$ (to be used if the t is not known)

where v is the instantaneous velocity, v_i is the initial velocity, v_f is the final velocity, x is the position, x_i is the initial position, x_f is the final position, and a is the acceleration.

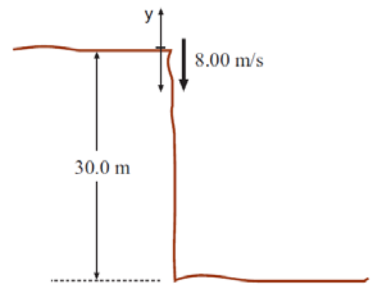
Section 2.9

<u>Free Fall</u>: If a body is falling freely without air friction, its acceleration, $a = g = 9.8$ (m/s²), is constant and its speed increases uniformly, but the distance moved increases non-uniformly. The equations of motion are given by:

$$v_f = v_i + gt, \quad v_f^2 = v_i^2 + 2gs, \quad d = v_i t + \frac{1}{2} gt^2$$

where the variables are defined earlier.

Example 1: A ball is thrown directly downward with an initial speed of 8.00 m/s from a height of 30.0 m. When does the ball strike the ground?



Ball is thrown straight down with speed of 8.00 $\frac{m}{s}$

Answer: We diagram the problem as in the figure. We have to choose a coordinate system, and here I will let the origin of the *y* axis be at the place where the ball starts its motion (at the top of the 30m height). With this choice, the ball starts its motion at $y_i = 0$ and strikes the ground when $y_f = -30$ m. We can now see that the problem is asking us: At what time does $y_f = -30$ m? We have $v_i = -8.00$ m/s (minus because the ball is thrown downward!) and the acceleration of the ball is $a = g = -9.8$ (m/s²), so at any time *t* the *y* coordinate is given by

$$y_f = y_i + v_i t + \frac{1}{2} g t^2 = 0 + (-8 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$$

But at the time of the impact we have

$$y_f = -30 \text{ m} = (-8 \text{ m/s})t + (-4.9 \text{ m/s}^2)t^2$$

an equation for which we can solve for *t*. We rewrite it as:

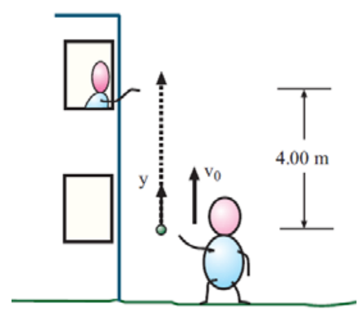
$$4.9t^2 + 8t - 30 = 0$$

which is just a quadratic equation in *t* and its solutions are:

$$t = \begin{cases} -3.42 \text{ s} \\ 1.78 \text{ s} \end{cases}$$

Our answer is one of these . . . which one? Obviously the ball had to strike the ground at some positive value of *t*, so the answer is $t = \underline{1.78 \text{ s}}$.
The ball strikes the ground 1.78 s after being thrown.

Example 2: A student throws a set of keys vertically upward to his brother in a window 4.00m above. The keys are caught 1.50 s later by the brother's outstretched hand.
(a) With what initial velocity were the keys thrown?
(b) What was the velocity of the keys just before they were caught?



Student throws her keys into the air

Answer:
(a)We draw a simple picture of the problem; such a simple picture is given in the figure. Having a picture is important, but we should be careful not to put too much into the picture; the problem did not say that the keys were caught while they were going up or going down. For all we know at the moment, it could be either one!
We will put the origin of the *y* axis at the point where the keys were thrown. This simplifies things in that the initial *y* coordinate of the keys is $y_i = 0$. Of course, since this is a problem about free–fall, we know the acceleration: $a = g = -9.8$ (m/s²).
What mathematical information does the problem give us? We are told that when $t = 1.50$ s, the y_f coordinate of the keys is $y_f = 4.00$ m. Is this enough information to solve the problem? We write the equation for $y(t)$:

$$y_f = y_i + v_i t + \frac{1}{2} g t^2 = v_i t + \frac{1}{2} g t^2$$

where v_i is presently unknown. At $t = 1.50$ s, $y_f = 4.00$ m, so:

$$4 = v_i t + \frac{1}{2} g t^2 = 1.5 v_i + \frac{1}{2} \left(-9.8\right)\left(1.5\right)^2$$

Now we can solve for v_i. Rearrange this equation to get:

$$v_i = 10 \frac{\text{m}}{\text{s}}$$

(b) We want to find the velocity of the keys at the time they were caught, that is, at t = 1.50 s. We know v_i; the velocity of the keys at all times follows from the equation:

$$v_f = v_i + at = v_i = 10 \frac{\text{m}}{\text{s}} - 9.8 \frac{\text{m}}{\text{s}^2} \times 1.5 \text{ s} = -4.68 \frac{\text{m}}{\text{s}}$$

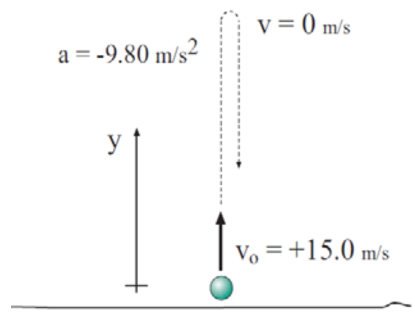
So the velocity of the keys when they were caught was −4.68 m/s . Note that the keys had a negative velocity; this tells us that the keys were moving downward at the time they were caught!

Example 3: A ball is thrown vertically upward from the ground with an initial speed of 15.0 m/s.

(a) How long does it take the ball to reach its maximum altitude?

(b) What is its maximum altitude?

(c) Determine the velocity and acceleration of the ball at t = 2.00 s.



Ball is thrown straight up with initial speed 15.0 $\frac{\text{m}}{\text{s}}$.

Answer:

(a) An illustration of the data given in this problem is given in the figure. We measure the coordinate y upward from the place where the ball is thrown so that y_i = 0. The ball's acceleration while in flight is $a = g = -9.8\,(\text{m/s}^2)$. We are given that v_i = +15.0 m/s . The ball is at maximum altitude when its (instantaneous) velocity v_f is zero (it is neither going up nor going down) and we can use the expression for v_f to solve for t:

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{v_f - v_i}{a}$$

Plug in the values for the top of the ball's flight and get:

$$t = \frac{0 - 15}{-9.8} = \underline{1.53 \text{ s}}$$

The ball takes 1.53 s to reach maximum height.

(b) Now that we have the value of t when the ball is at maximum height we can plug it into $y_f = y_i + v_i t + \frac{1}{2} g t^2$ and find the value of $y = \left(y_f - y_i\right)$ at this time and that will be the value of the maximum height. But we can also use $v_f^2 = v_i^2 + 2ay$ since we know all the values except for y. Solving for y we find:

$$v_f^2 = v_i^2 + 2ay \quad \Rightarrow \quad y = \frac{v_f^2 - v_i^2}{2a}$$

Plugging in the numbers, we get

$$y = \frac{0^2 - 15^2}{2(-9.8)} = \underline{11.5 \text{ m}}$$

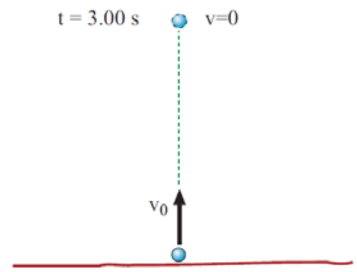
The ball reaches a maximum height of 11.5 m.

(c)At t = 2.00 s (that is, 2.0 seconds after the ball was thrown), using the equation $v_f = v_i + at$ to find:

$$v_f = 15.0 + (-9.80)(2.00) = \underline{-4.60 \ \frac{\text{m}}{\text{s}}}$$

so at t = 2.00 s the ball is on its way back down with a speed of 4.60 (m/s) .

Example 4: A baseball is hit such that it travels straight upward after being struck by the bat. A fan observes that it requires 3.00 s for the ball to reach its maximum height. Find
(a) its initial velocity and
(b) its maximum height. Ignore the effects of air resistance.



Ball is hit straight up; reaches maximum height 3.00 s later.

Answer:
(a) An illustration of the data given in the problem is given in above figure. For the period from when the ball is hit to the time it reaches maximum height, we know the time interval, the acceleration ($a = -g$) and also the final velocity, since at maximum height the velocity of the ball is zero. Then $v_f = v_i + at$ gives us v_i :

$$\Rightarrow \quad v_i = 0 - (-9.80)(3.00) = \underline{29.4 \ \frac{\text{m}}{\text{s}}}$$

The initial velocity of the ball was +29.4 m/s .

(b) To find the value of the maximum height, we need to find the value of the y coordinate at time t = 3.00 s. We can use:

$$v_f^2 = v_i^2 + 2a(y_f - y_i) \quad \Rightarrow \quad (y_f - y_i) = \frac{v_f^2 - v_i^2}{2a}$$

Plugging in the numbers we find that the change in y coordinate for the trip up was:

$$(y_f - y_i) = \frac{v_f^2 - v_i^2}{2a} = \frac{0^2 - 29.4^2}{2(-9.8)} = \underline{44.1 \text{ m}}$$

The ball reached a maximum height of 44.1m.

Challenging problem: A falling object requires 1.50 s to travel the last 30.0m before hitting the ground. From what height above the ground did it fall?

Answer:

This is an intriguing sort of problem... very easy to state, but not so clear as to where to begin in setting it up!
The first thing to do is draw a diagram. We draw the important points of the object's motion, as in the figure. The object has zero velocity at A; at B it is at a height of 30.0m above the ground with an unknown velocity. At C it is at ground level, the time is 1.50 s later

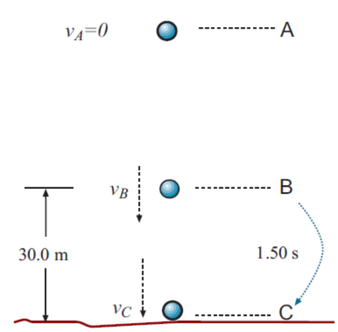


Diagram for the falling object

than at *B* and we also don't know the velocity here. Of course, we know the acceleration: $a = -9.80\ \text{m/s}^2$!!

We are given all the information about the trip from *B* to *C*, so why not try to fill in our knowledge about this part? We know the final and initial coordinates, the acceleration and the time so we can find the initial velocity (that is, the velocity at *B*). Let's put the origin at ground level; then, $y_0 = 30.0\text{m}$, $y = 0$ and $t = 1.50$ s, and using

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

we find:

$$v_0 t = (y - y_0) - \tfrac{1}{2} a t^2 = (0 - (30.0\,\text{m})) - \tfrac{1}{2}(-9.80\,\tfrac{\text{m}}{\text{s}^2})(1.50)^2 = -19.0\,\text{m}$$

so that

$$v_0 = \frac{(-19.0\,\text{m})}{t} = \frac{(-19.0\,\text{m})}{(1.50\,\text{s})} = -12.5\,\frac{\text{m}}{\text{s}}\ .$$

This is the velocity at point *B*; we can also find the velocity at *C* easily, since that is the final velocity, *v*:

$$v = v_0 + at = (-12.5\,\tfrac{\text{m}}{\text{s}}) + (-9.80\,\tfrac{\text{m}}{\text{s}^2})(1.50\,\text{s}) = -27.3\,\tfrac{\text{m}}{\text{s}}$$

Now we can consider the trip from the starting point, *A* to the point of impact, *C*. We don't know the initial *y* coordinate, but we do know the final and initial velocities: The initial velocity is $v_0 = 0$ and the final velocity is $v = -27.3$ m/s, as we just found. With the origin set at ground level, the final *y* coordinate is $y = 0$. We don't know the time for the trip, but if we use:

$$v^2 = v_0^2 + 2a(y - y_0)$$

we find:

$$(y - y_0) = \frac{(v^2 - v_0^2)}{2a} = \frac{(-27.3\,\frac{\text{m}}{\text{s}})^2 - (0)^2}{2(-9.80\,\frac{\text{m}}{\text{s}^2})} = -38.2\,\text{m}$$

and we can rearrange this to get:

$$y_0 = y + 38.2\,\text{m} = 0 + 38.2\,\text{m} = 38.2\,\text{m}$$

and the so the object started falling from a height of 38.2m.

There are probably cleverer ways to do this problem, but here I wanted to give you the slow, patient approach!

Phys101 – Chapter 2 – Instructor: Dr. Ali M. Al –Shukri

Selected problems from the textbook

18. The position of a particle is given by $x = 20\,t - 5\,t^3$, where x is in meters and t is in seconds,

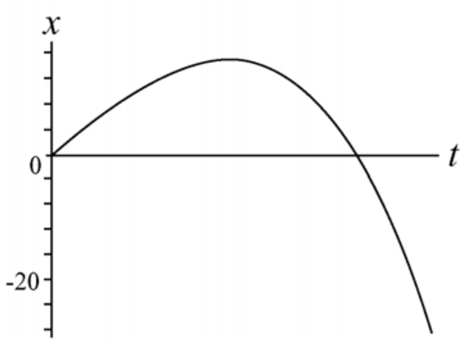
(a) When, if ever, is the particle's velocity v zero?
(b) When is its acceleration a zero?
(c) For what time range is a negative?
(d) For what time range is a positive?
(e) Graph $x(t)$, $v(t)$, and $a(t)$.

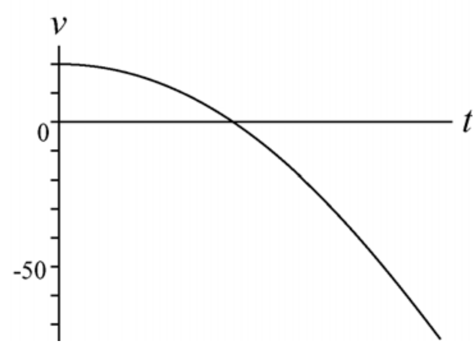
We can get $v(t)$ and $a(t)$ by differentiating $x(t) = 20\,t - 5\,t^3$.

$v(t) = dx/dt = 20 - 15\,t^2$ and

$a(t) = dv/dt = d^2x/dt^2 = -30\,t$

where x is in m, v in m/s, a in m/s^2, and t is in s.





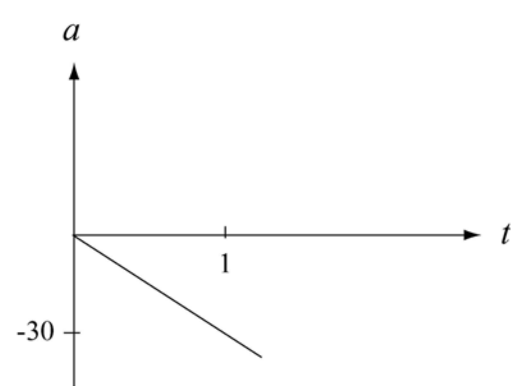
(a) From $0 = 20 - 15\,t^2$, the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$ ➔ $a(0) = 0$ (it vanishes at $t = 0$).

(c) The acceleration $a(t) = -30t$ is negative for $t > 0$

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) See the graphs Up and to the left



33. A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later.

(a) What is the magnitude of the car's constant acceleration before impact?

(b) How fast is the car traveling at impact?

(a) Since a is constant, the following equation can be used:

$x = x_0 + v_0 t + \frac{1}{2} a t^2 \;\Rightarrow\; a = 2(x - x_0 - v_0 t)/t^2$

If we take $x_0 = 0$ then $a = 2(x - v_0 t)/t^2$

Substituting $x = 24.0$ m, $v_0 = 56.0$ km/h = 15.55 m/s and $t = 2.00$ s, we find

$$a = \frac{2\left(24.0\,\text{m} - \left(15.55\,\text{m/s}\right)\left(2.00\,\text{s}\right)\right)}{\left(2.00\,\text{s}\right)^2} = -3.56\,\text{m/s}^2,$$

The negative sign is used to indicate that the acceleration is opposite to the direction of motion of the car (The car is slowing down).

(b) Since a is constant, the following equation can be used

$$v = v_0 + at$$

Substituting $a = -3.56$ m/s^2, $v_0 = 56.0$ km/h = 15.55 m/s, and $t = 2.00$ s, we find

$v = 15.55 + (-3.56)\times 2.00 = 15.55 - 7.12 = 8.43$ m/s
$= 8.43$ (m/s)$\times$(1 km/1000 m)$\times$(3600 s/1 h)
$= 30.348 = 30.3$ km/h

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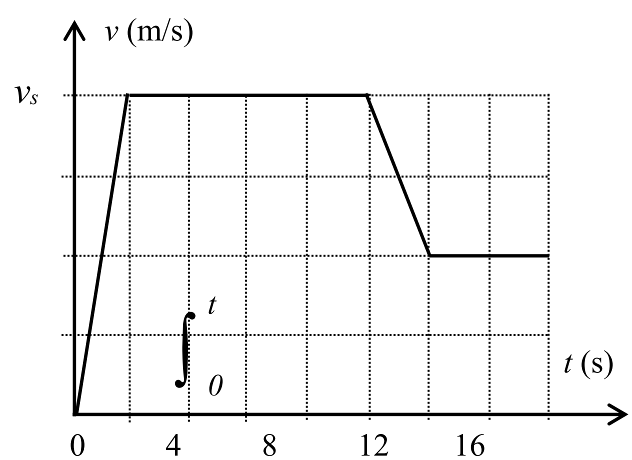
67. How far does the runner whose velocity – time graph is shown in the Figure travel in 16 s ? The figure's vertical scaling is set by $v_s = 8.0$ m/s.



Velocity = change in position over time interval

$v = dx/dt \;\Rightarrow\; dx = v\,dt \;\Rightarrow\; x - x_0 = \Delta x = \int_0^t v\,dt$

In case of a graph: Δx = area under $v(t)$ curve

Then distance traveled in time interval $t = 0$ to $t = 16$ s

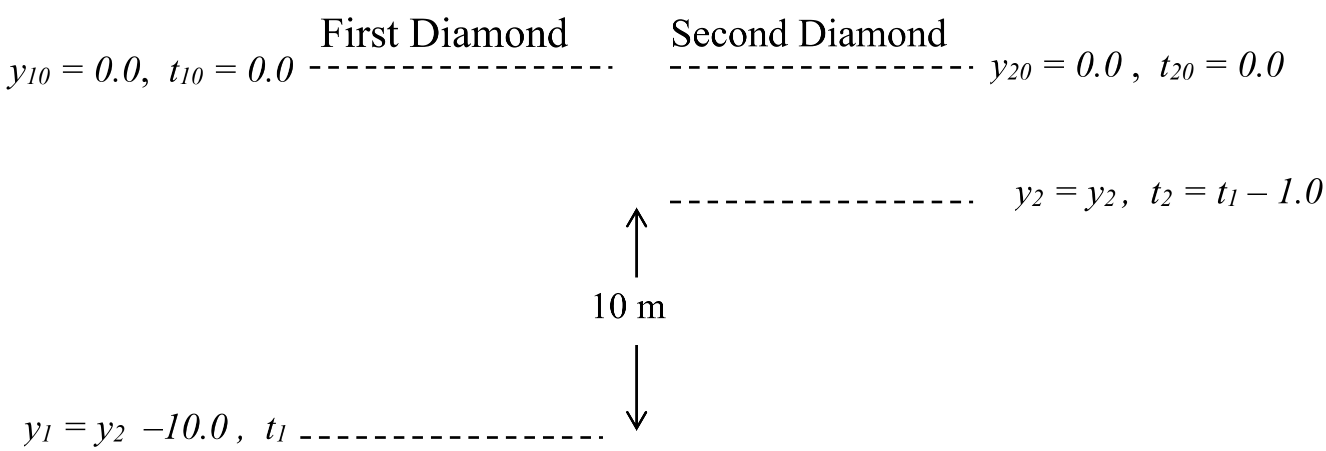
Δx = Area (t = 0 → 2) + Area (t = 2 → 10) +
Area (t = 10 → 12) + Area (t = 12 → 16)

Δx = area of a triangle + area of a rectangle + (areas of a triangle + and a rectangle) + area of a rectangle
= ½×(2 –0)×(8 –0) + (10 –2)×(8 –0) + [½×(12 –10)×(8 –4) + (12 –10)×(4 – 0)] + (16 –12)×(4 –0)
= 8 + 64 + [4 + 8] + 16 = 100 m

---

76. Two diamond begin a free fall from rest from the same height, 1.0 s apart. How long after the first diamond begins to fall will they will be 10.0 m apart?

Using the free fall equations: $y = y_0 + v_0 t - \frac{1}{2} g t^2$
See the diagram below for explanation of the problem
Note: $y_{10} = y_{20} = 0.0$, $t_{10} = t_{20} = 0.0$, $t_2 = t_1 - 1.0$, and $v_{10} = v_{20} = 0.0$



$y_1 = -\frac{1}{2} g t_1^2$, $y_2 = -\frac{1}{2} g t_2^2$, $y_1 = y_2 - 10$, and $t_2 = t_1 - 1$
$y_1 = -\frac{1}{2} g t_1^2 = y_2 - 10 = -\frac{1}{2} g t_2^2 - 10 = -\frac{1}{2} g (t_1 - 1)^2 - 10.0$
$-\frac{1}{2} g t_1^2 = -\frac{1}{2} g (t_1 - 1)^2 - 10 \Rightarrow -\frac{1}{2} g t_1^2 = -\frac{1}{2} g t_1^2 + g t_1 - \frac{1}{2} g - 10$
$g t_1 - \frac{1}{2} g = 10 \Rightarrow t_1 = (10 + \frac{1}{2} g)/g = (10+4.9)/9.8 = 14.9/9.8 = 1.52$ s
$t_1 = 1.5$ s.

PHYS101 - Chapter 2 (Instructor: Dr. Ali Al-Shukri)

Extra Problems (Chapter 2):

2. Two cars A and B travel on a straight line. The displacement of car A is given by $x_A(t) = 2.60\ t + 1.20\ t^2$, where t is in seconds and x_A in m. The displacement of car B is given by $x_B(t) = 2.80\ t^2 - 0.20\ t^3$. At what time the two cars will have the same acceleration?

   a. 2.67 s b. 6.27 s c. 7.26 s d. 9.36 s e. 0.67 s

3. A ball is thrown from ground straight upward with a velocity of 26 m/s. How long does it take the ball to strike the ground?

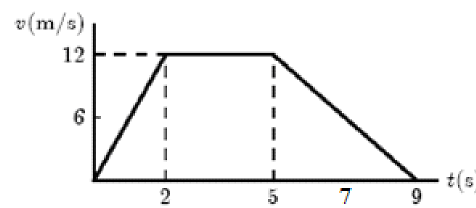
   a. 5.3 s b. 2.7 s c. 1.6 s d. 0.8 s e. 7.5 s

4. A ball is thrown from ground straight upward. If it stays in flight for 6.0 s, find the initial velocity and the maximum height it reaches. Also find the average velocity for the entire flight.

   a. v_0 = 29.4 m/s , h = 44.1 m , v_{ave} = 0

5. Two automobiles, 150 kilometers apart, are traveling toward each other. One automobile is moving at 60 km/h and the other is moving at 40 km/h. In how many hours will they meet?

   a. 1.5 b. 2.0
   c. 1.0 d. 2.5 e. 3.0



6. The graph shown in the Figure represents the straight-line motion of a car. Find its acceleration at t = 6 s.

   a. –3.0 m/s² b. +5.0 m/s² c. +3.0 m/s² d. –5.0 m/s²

7. A car travels along a straight line at a constant velocity of 18 m/s for 2.0 s and then accelerate at − 6.0 m/s² for a period of 3.0 s. The average velocity of the car during the whole 5.0 s is:

a. 13 m/s b. 18 m/s c. 10 m/s d. 16 m/s

8. The velocity as a function of time for a particle moving along the x-axis is shown in Figure 1. The motion clearly has two different parts: the first part is from t = 0 to t = 2.0 s, and the second part is from t = 2.0 s to t = 6.0 s. Which one of the following statements is correct?

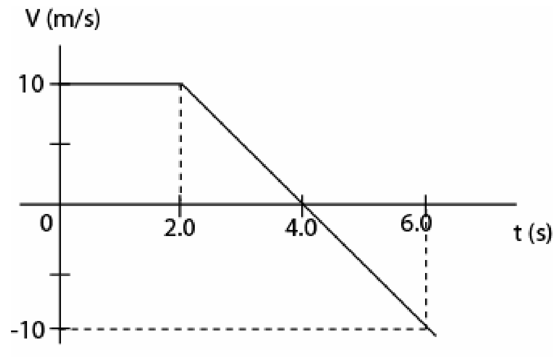


Figure 1

a. At t = 4.0 s the acceleration is -5.0 m/s^2
b. At t = 4.0 s the acceleration is zero
c. From t =0 to t=6.0 s, the displacement is zero
d. From t =0 to t=6.0 s, the displacement is -20 m
e. At t = 1.0 s the acceleration is 10 m/s^2

9. A particle moves along the x axis. Its position is given by the equation x = 2.0 + 3.0 t − t^3 with x in meters and t in seconds. The average acceleration from t = 0 to t = 2.0 s is:

a. −6.0 m/s^2 b. 3.0 m/s^2 c. −2.0 m/s^2 d. 4.0 m/s^2

10. An arrow is shot straight up with an initial speed of 98 m/s. If friction is neglected, how high the arrow can reach?

a. 490 m b. 980 m c. 250 m d. 98 m

11. A stone is thrown vertically downward from the top of a 40 m tall building with an initial speed of 1.0 m/s. After 2.0 s the stone will have traveled a distance of

a. 22 m b. 38 m c. 40 m d. 25 m e. 15 m

12. A stone and a ball are thrown vertically upward with different initial speeds: 20 m/s for the stone and 10 m/s for the ball. If the maximum height reached by the ball is H then the maximum height reached by the stone is:

a. 4 H b. 2 H c. H d. H/2 e. H/4

13. A particle starts from the origin at t = 0 and moves along the positive x-axis. A graph of the velocity of the particle as a function of time is show in Figure 1. The average velocity of the particle between t = 0.0 s and 5.0 s is:

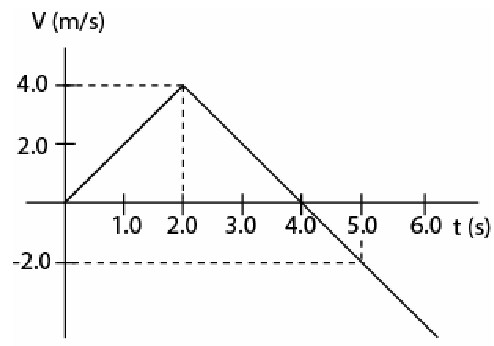


Figure 1

a. 1.4 m/s b. 1.0 m/s c. -2.0 m/s d. 0 m/s

14. At a traffic light, a truck traveling at 10 m/s passes a car as it starts from rest. The truck travels at a constant velocity and the car accelerates at 4.0 m/s^2. How much time does the car take to catch up with the truck?

a. 5.0 s b. 2.0 s c. 15 s d. 20 s e. 25 s

15. The coordinate of a particle in meters is given by $x(t) = 2.0\,t - 2.0\,t^2$, where the time t is in seconds. The particle is momentarily at rest at time t equal to:

a. 0.50 s b. 0.75 s c. 2.0 s d. 1.3 s

16. An object starts from rest at the origin and moves along the x-axis with a constant acceleration of 4 m/s^2. Its average velocity as it goes from x = 2 m to x = 18 m is:

a. 8 m/s b. 2 m/s c. 6 m/s d. 5 m/s

17. Two cars are a distance d apart and traveling toward each other. One car is moving at 60. km/h and the other is moving at 40. km/h. If they meet after 90 minutes of traveling, find the distance d.

a. 150 km b. 120 km c. 100 km d. 50 km

18. A helicopter at height h (m) from the surface of the sea is descending at a constant speed of v (m/s). The time it takes to reach the surface of the sea can be found from:

a. $-h = -v\,t$ b. $h = \frac{1}{2} g\,t^2$ c. $-h = \frac{1}{2} g\,t^2$
d. $h = v\,t - \frac{1}{2} g\,t^2$ e. $-h = -v\,t - \frac{1}{2} g\,t^2$

19. A particle starts from rest at t = 0 s. Its acceleration as a function of time is shown in Fig. 1. What is its speed at the end of the 6.0 s?

a. 4.0 m/s b. 0 m/s c. 12 m/s d. 2.0 m/s e. -12 m/s

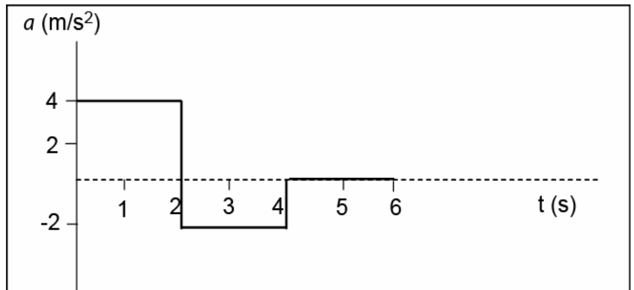


Fig. 1

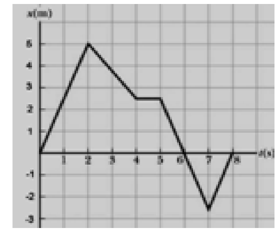
20. The position of a particle x(t) as a function of time (t) is described by the equation: $x(t) = 2.0 + 3.0\,t - t^3$, where x is in m and t is in s. What is the maximum positive position of the particle on the x-axis?

a. 4.0 m b. 3.0 m c. 2.0 m d. 1.0 m

21. A stone is thrown vertically downward from a building with an initial speed of 2.0 m/s. It reaches the ground after 5.0 s. What is the height of the building?

a. 130 m b. 60 m c. 180 m d. 120 m

Chapter 2, selected problems



Q1. The position versus time for a certain particle moving along the x-axis is shown in **Figure**. The average velocity in the time interval 4.0 s to 7.0 s is: A)– 1.7 m/s

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Q2. A ball is thrown directly downward from a height of 30.0 m. It takes 1.79 s to reach the ground. Find the magnitude of the initial velocity. A) 7.99 m/s

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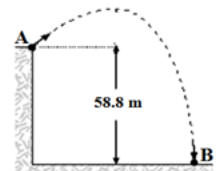

Q3. A stone is thrown outward from point A at the top of a 58.8 m high cliff with an upward velocity component of 19.6 m/s (see **Figure**). Assume that it lands on the ground, at point B, below the cliff, and that the ground below the cliff is flat. How long was the stone in the air? [Neglect the air resistance]. A) 6.00 s

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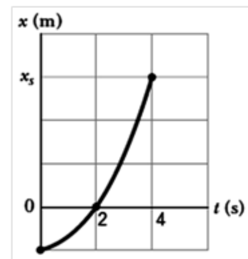

Q4. Figure 3 illustrates the motion of a particle starting from rest and moving along an x-axis with a constant acceleration. The figure's vertical scaling is set by $x_s = 12$ m. The particle's acceleration is A) 2.0 m/s^2

Q6. A stone is thrown vertically upwards with an initial speed of 4.0 m/s from a window which is 8.0 m above the ground. With what speed will the stone hit the ground? (Neglect air resistance) A) 13 m/s

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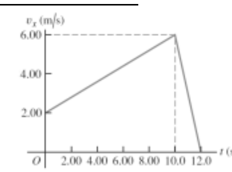

Q6. A man is running in a straight line (along the x-axis). The graph in **Figure** shows the man's velocity as a function of time. During the first 12.0 s, the total distance traveled is A) 46.0 m

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Q7. A rock is thrown vertically upward from ground level at time t =0.0 s. At t =1.5 s it passes the top of a tall tower, and then 1.0 s later it reaches its maximum height. What is the height of the tower? A) 26 m

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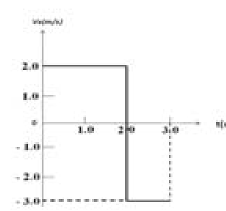

Q8: A ball moves in a straight line along the x-axis and **Figure** shows its velocity as a function of time t. What is the ball average velocity and average speed, respectively, over a period of 3.00 s. A) 0.330 m/s, 2.33 m/s

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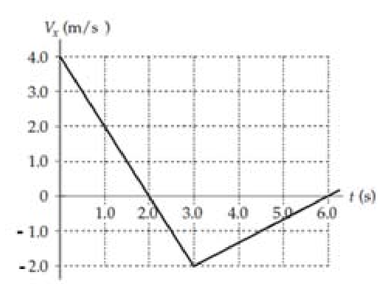
Q9: A car travels in a straight line. First, it starts from rest at point A and accelerates at a rate of 5.00 m/s^2 until it reaches a speed of 100 m/s at point B. The car then slows down at a constant rate of 8.00 m/s^2 until it stops at point C. Find the time the car takes for this trip (from point A to point C). **A) 32.5 s**



Q10: Figure 1 shows the velocity V_x (m/s) of a particle moving along the x-axis. If x = 2.0 m at t = 1.0 s, what is the position, measured in meters, of the particle at t = 6.0 s? A) –1