Chapter 14

FLUIDS

1. What is a Fluid?

- A **fluid** is a substance that can flow.
- Fluids conform to the boundaries of any container in which we put them.

2. Density and Pressure

- When we discuss rigid bodies, we found that the physical quantities of mass and force are useful for studying such bodies.
- In the case of fluids, it is more useful to speak of **density** and **pressure**.
- **Density**:

  The density ρ of a fluid at any point is the ratio of the mass Δm of an isolated volume element ΔV around that point to the volume element ΔV:

$$\rho = \frac{\Delta m}{\Delta V}.$$

2. Density and Pressure

A fluid sample is large relative to atomic dimensions. We then write

$$\rho = \frac{m}{V}.$$

where m and V are the sample's volume and mass, respectively.

Density is a scalar quantity. Its SI unit is $\mathrm{kg/m^2}$.

The density of a gas depends largely on the pressure, unlike liquids. That is because gasses are readily compressible, unlike liquids.

2. Density and Pressure

- **Pressure**:

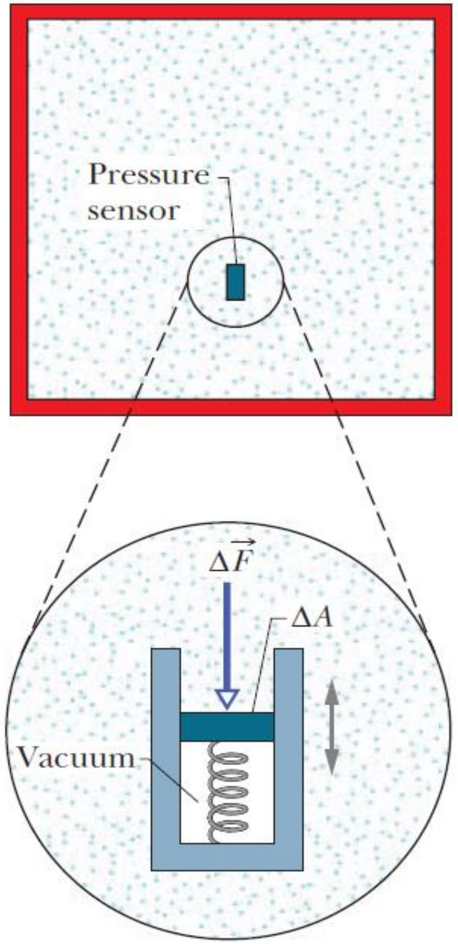
  Consider the small pressure sensor in the figure. The fluid exerts a force ΔF normal to the piston of surface area ΔA. The pressure on the piston is defined as

$$P = \frac{\Delta F}{\Delta A}.$$

  If the force is uniform over a flat area A, we can write

$$P = \frac{F}{A},$$

  where F is the magnitude of the normal force on area A.



2. Density and Pressure

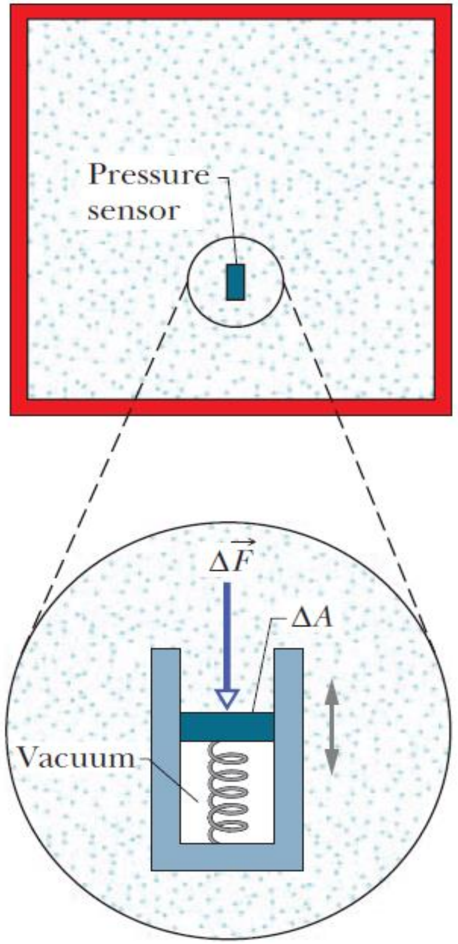
- **Pressure**:

  Pressure is a scalar quantity. The expression for pressure includes the magnitude of the force!

  The SI unit of pressure is $\mathrm{N/m^2}$, which is given the special name **pascal** (Pa).

  Another common unit is **atmosphere** (atm), the approximate average pressure of the atmospheric pressure at sea level, given as

$$1\ \mathrm{atm} = 1.01\ \times 10^5 \mathrm{Pa} = 760\ \mathrm{torr\ (mm\ Hg)}.$$



2. Density and Pressure

Example 1: A living room has floor dimensions of $3.5\ \mathrm{m}$ and $4.2\ \mathrm{m}$ and a height of $2.4\ \mathrm{m}$.

(a) What does the air in the room weigh when the air pressure is $1.0\ \mathrm{atm}$?

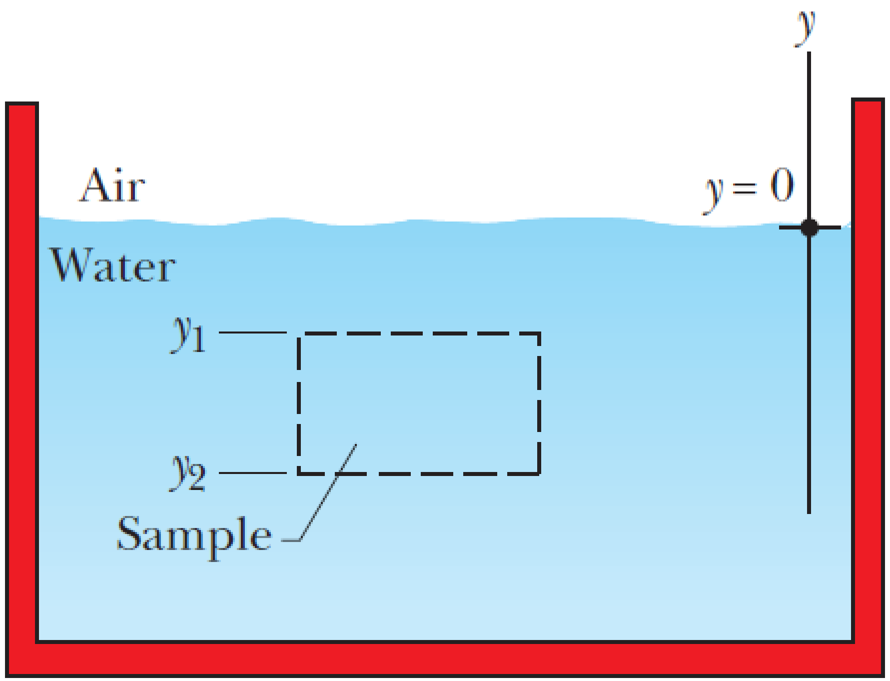
$$\begin{aligned} mg &= (\rho V)g \\ &= (1.21\ \mathrm{kg/m^3})(3.5\ \mathrm{m} \times 4.2\mathrm{m} \times 2.4\ \mathrm{m})(9.8\ \mathrm{m/s^2}) \\ &\approx 420\ \mathrm{N}. \end{aligned}$$

(b) What is the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of $0.040\ \mathrm{m}^2$?

$$F = pA = \left(1\ \mathrm{atm} \times \frac{1.01\ \times 10^5\ \mathrm{N/m^2}}{1\ \mathrm{atm}}\right)(0.040\ \mathrm{m^2}) = 4.0\ \times 10^3\ \mathrm{N}.$$

3. Fluids at Rest

- The pressures due to fluids that are static (at rest) are usually called *hydrostatic* pressures.
- Here we want to find an expression for hydrostatic pressure as a function of depth or altitude.
- Consider an imaginary cylinder of water with base area A and y_1 and y_2 are the heights of its two faces.
- The water sample is in static equilibrium; it is stationary and the forces on it balance.



3. Fluids at Rest

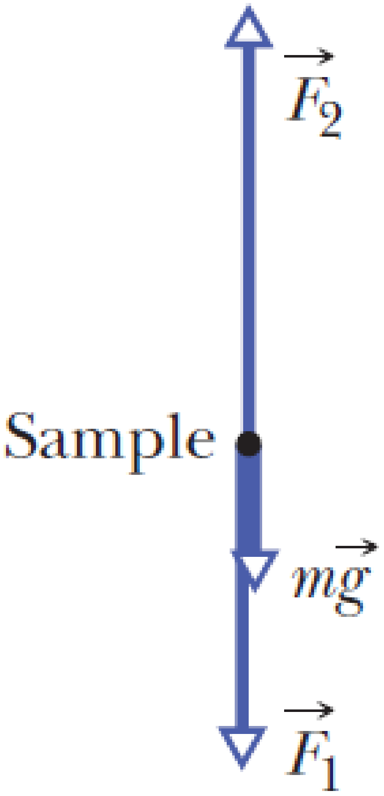
- The balance of these forces is written as

$$F_2 = F_1 + mg.$$

- We then use that $F_1 = p_1 A$, $F_2 = p_2 A$ and $m = \rho V = \rho A(y_1 - y_2)$ and write

$$p_2 = p_1 + \rho g(y_1 - y_2).$$

- This equation can be used to find pressure both in a liquid as a function of depth, and in the atmosphere as a function of altitude or height.

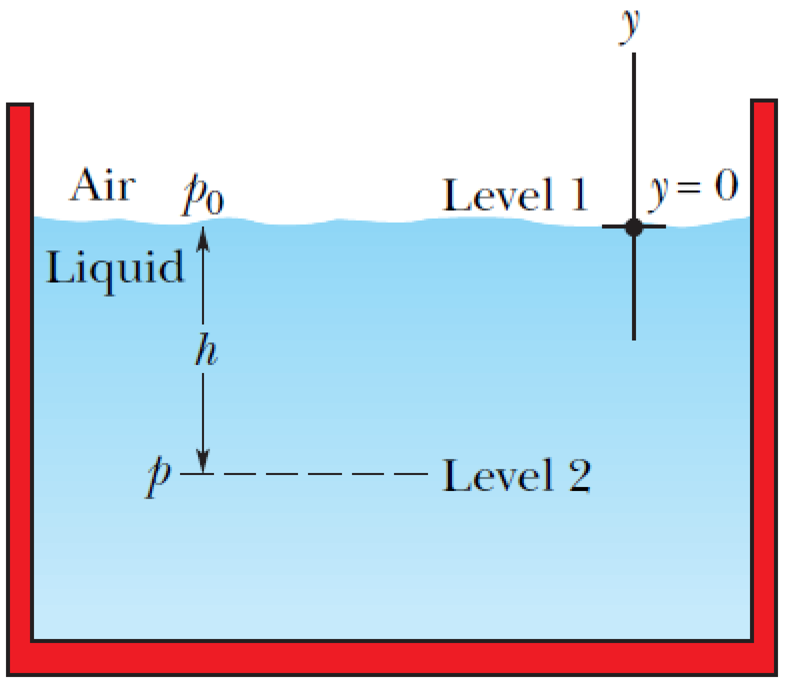


3. Fluids at Rest

- To find pressure p in a liquid as a function of depth, we use set $y_1 = 0$, $p_1 = p_0$, $y_2 = -h$, and $p_2 = p$ in the previous equation, which then becomes

$$p = p_0 + \rho g h.$$

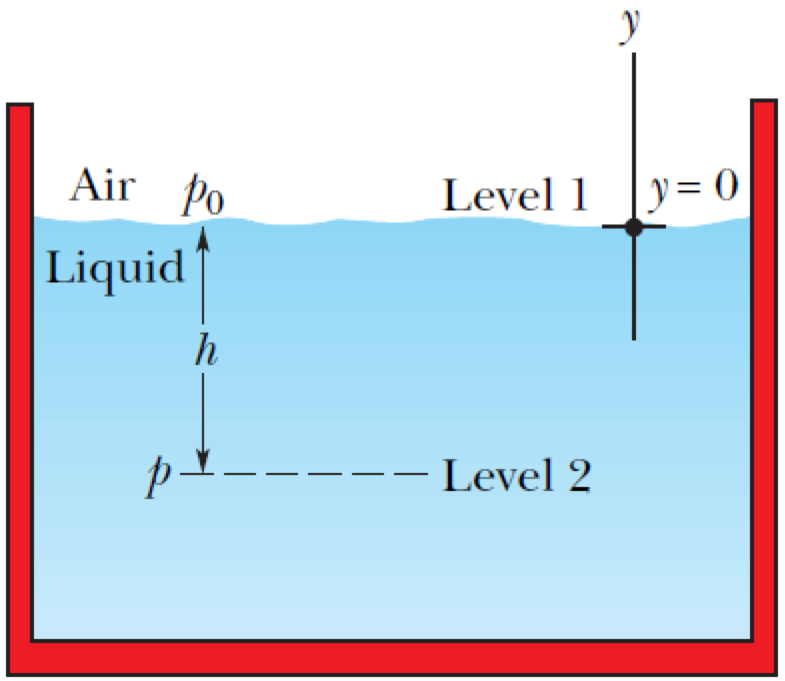
- The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.



3. Fluids at Rest

- p is said to be the total pressure, or **absolute pressure.** The difference between an absolute pressure $p - p_0 = \rho g h$ and an atmospheric pressure is called the **gauge pressure.**
- To find pressure p in the atmosphere as a function of altitude or height we set $y_1 = 0$, $p_1 = p_0$, $y_2 = d$, and $p_2 = p$ to get

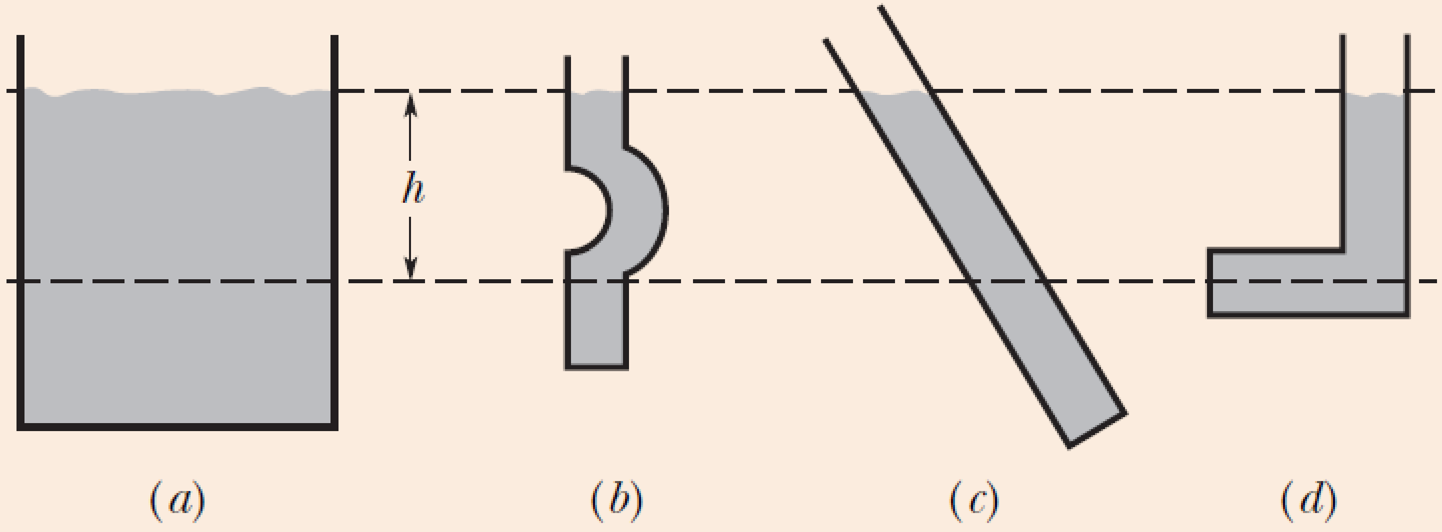
$$p = p_0 - \rho_{\text{air}}\, g d.$$



3. Fluids at Rest

CHECKPOINT 1

The figure shows four containers of olive oil. Rank them according to the pressure at depth h, greatest first.



All tie

3. Fluids at Rest

Example 2: A novice scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface. He ignores instructions and fails to exhale during his ascent. When he reaches the surface, the difference between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start? What potentially lethal danger does he face?

Using

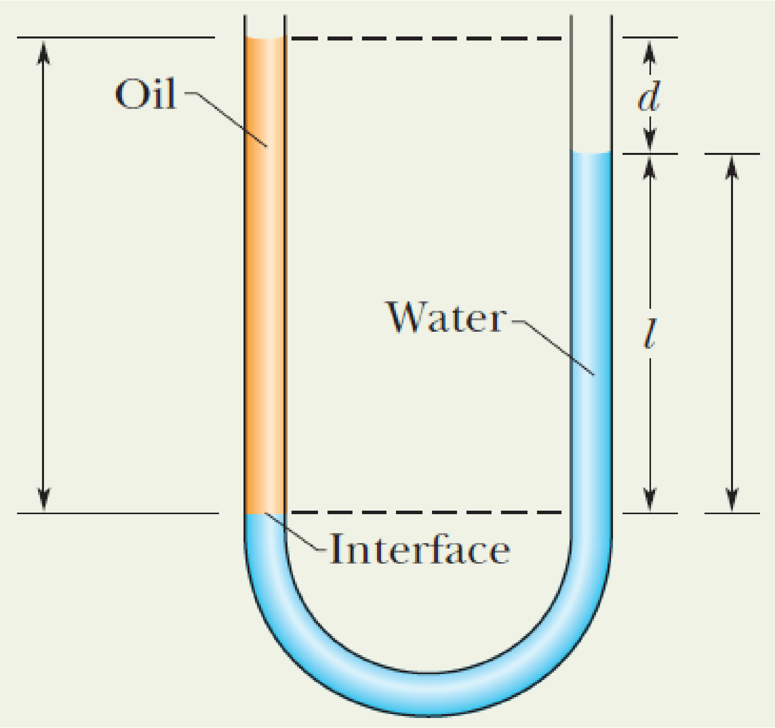
$$p = p_0 + \rho g h,$$

we can relate $\Delta p = p - p_0$ to h. Solving for h and substituting

$$h = \frac{\Delta p}{\rho g} = \frac{9300\ \mathrm{Pa}}{\left(998\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 0.95\ \mathrm{m}.$$

3. Fluids at Rest

Example 3: The U-tube in the figure contains two liquids in static equilibrium: Water of density $\rho_w = 998\,\mathrm{kg/m^3}$ is in the right arm, and oil of unknown density ρ_x is in the left. Measurement gives $l = 135\,\mathrm{mm}$ and $d = 12.3\,\mathrm{mm}$. What is the density of the oil?



Both fluid columns produce the same pressure p_{int} at the level of the interface. We therefore write

$$p_{\mathrm{int}} = p_0 + \rho_w g l \quad \text{(right arm)},$$

and

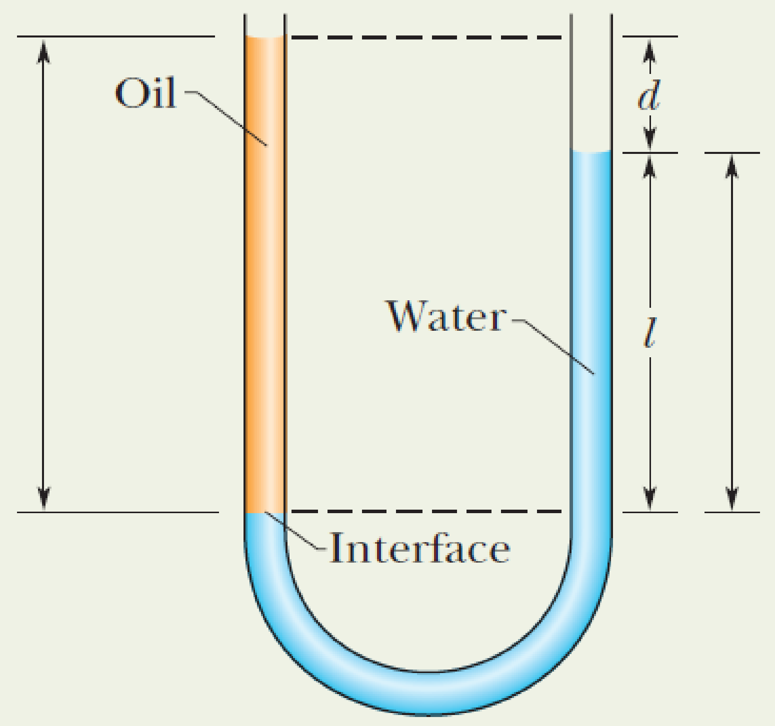
$$p_{\mathrm{int}} = p_0 + \rho_x g(l + d) \quad \text{(left arm)},$$

3. Fluids at Rest

Solving for d we find that

$$\rho_x = \rho_w \frac{l}{l + d} = (998 \text{ kg/m}^3) \frac{135 \text{ mm}}{135 \text{ mm} + 12.3 \text{ mm}}$$

$$= 915 \frac{\text{kg}}{\text{m}^3}.$$



4. Measuring Pressure

- <u>The Mercury Barometer</u>

  The figure shows a very basic *mercury barometer*. We can find the atmospheric pressure p_0 in terms of the height h of the mercury column. Substituting

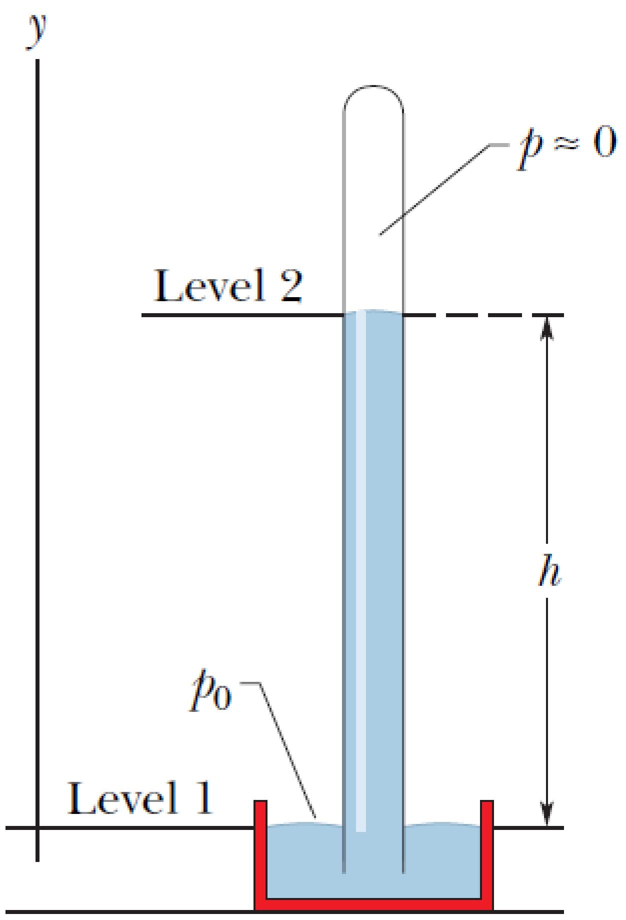
  $$y_1 = 0,\ p_1 = p_0 \text{ and } y_2 = h,\ p_2 = 0,$$

  into the equation $p_2 = p_1 + \rho g(y_1 - y_2)$, gives that

  $$p_0 = \rho g h,$$

  where ρ is the density of mercury.

  Note that the height of the mercury column depends on g (which depends on location) and ρ (which depends of temperature).



4. Measuring Pressure

- <u>The Open-Tube Manometer</u>

  The figure shows an *open-tube manometer*. We can find the gauge pressure p_g in terms of the height h. We substitute

$$y_1 = 0,\ p_1 = p_0 \text{ and } y_2 = -h,\ p_2 = p,$$

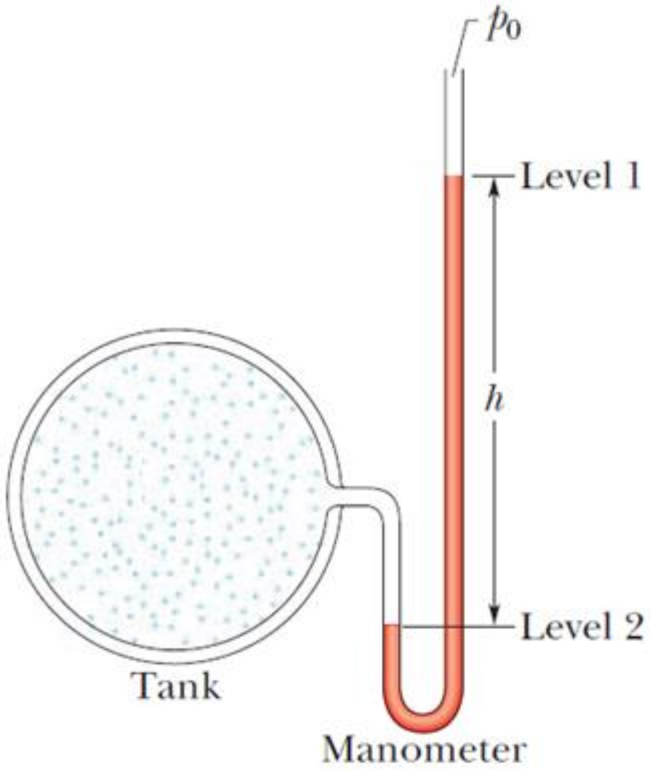
into the equation $p_2 = p_1 + \rho g(y_1 - y_2)$ and get

$$p = p_0 + \rho g h,$$

or

$$p_g = p - p_0 = \rho g h,$$

where ρ is the density of the liquid in the tube.

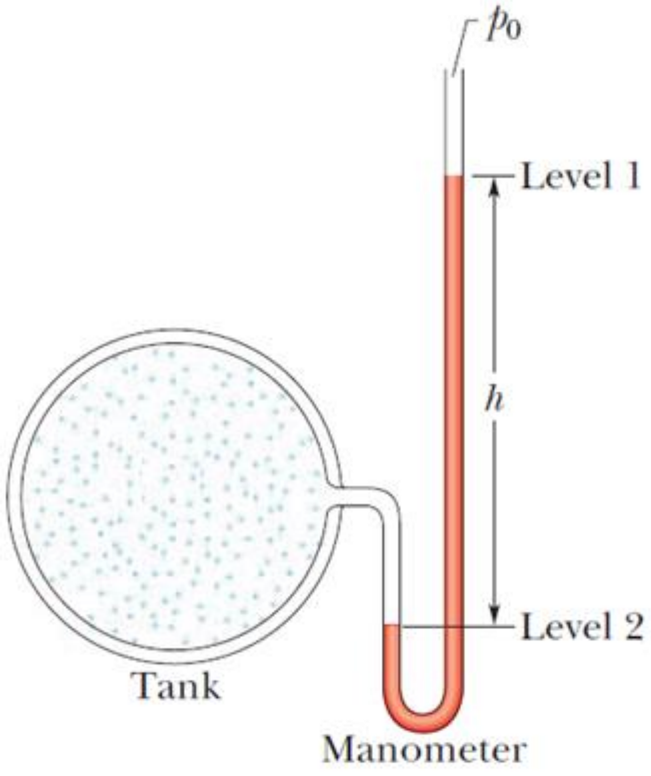


4. Measuring Pressure

- The Open-Tube Manometer

  p_g can be positive or negative, depending on whether $p > p_0$ or $p < p_0$.

  For example, p_g is positive in a tire and negative in a light bulb.

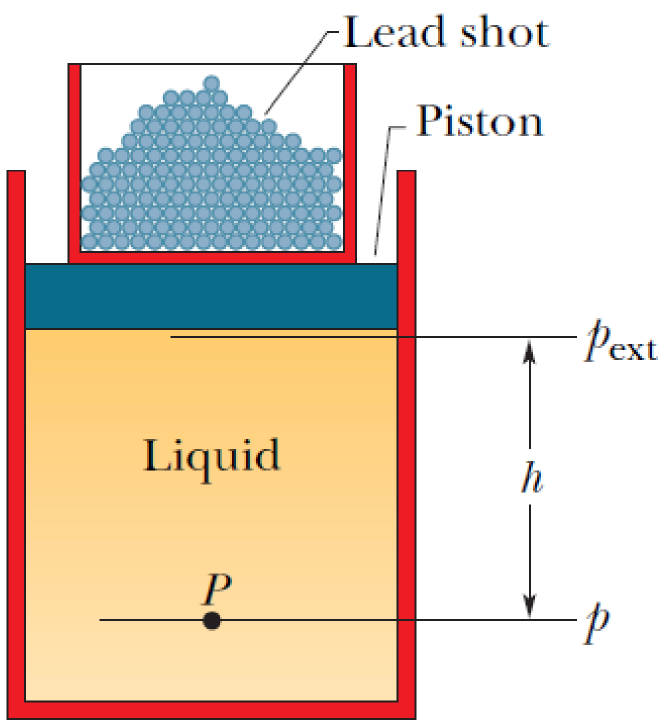


5. Pascal's Principle

- Pascal stated that: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.
- Demonstrating Pascal’s Principle

  Consider the system in the figure. The atmosphere, container, and shot exert pressure p_{ext} on the piston and the liquid. The pressure p at any point P in the liquid is

$$p = p_{ext} + \rho h g.$$



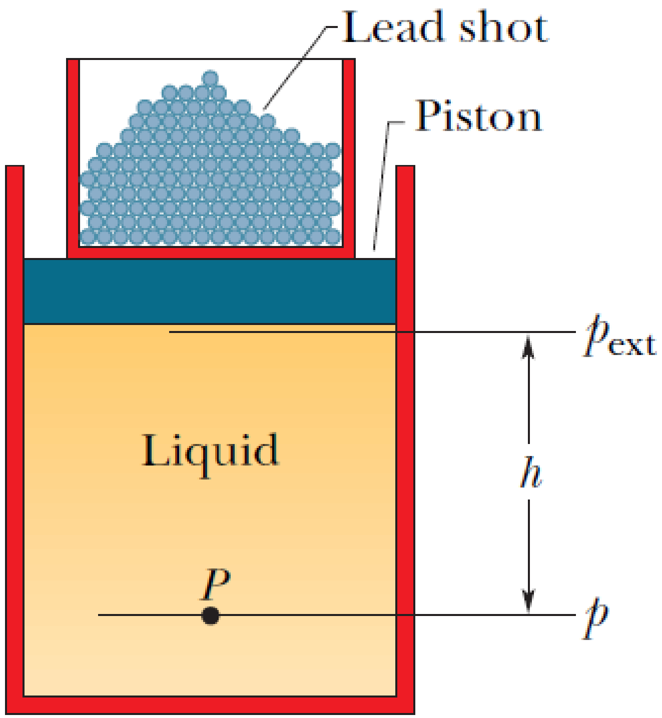
5. Pascal's Principle

- Demonstrating Pascal's Principle

  If you add more lead shot to the container p_{ext} is increased by Δp_{ext}. The pressure change at P is

  $$\Delta p = \Delta p_{ext}.$$

  The pressure change is independent of h, so it holds for all points within the liquid, as Pascal's principle states.

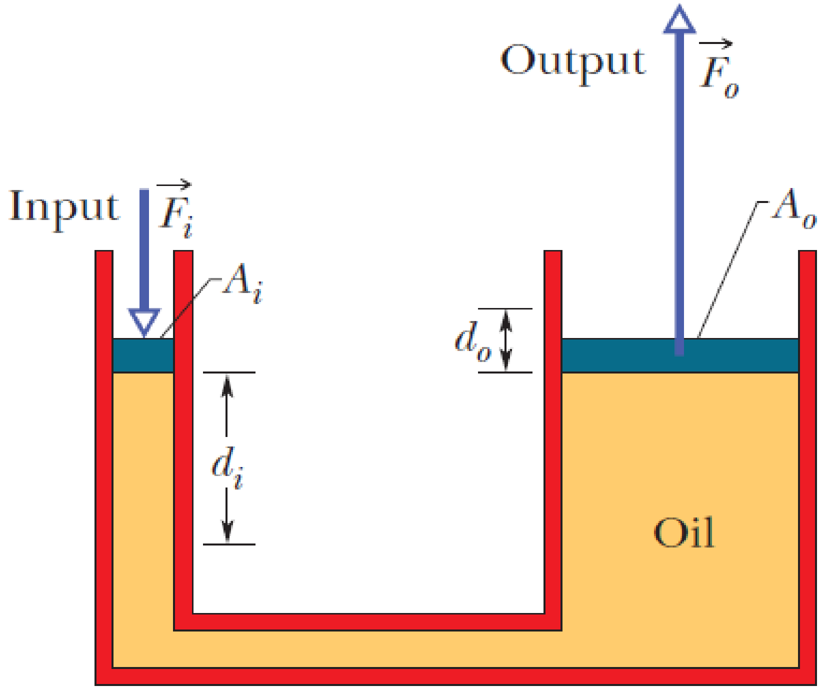


5. Pascal's Principle

- Pascal's Principle and the Hydraulic Lever

The figure shows a hydraulic lever. A downward force $\vec{F}_i$ is applied on the left-hand piston of area A_i. The oil then produces an upward force of magnitude F_o on the right-hand piston of area A_o. To keep the system in equilibrium, there must be a downward force of magnitude F_o on the output piston from an external load (now shown). The applied force $\vec{F}_i$ and the downward force $\vec{F}_o$ from the load produce a change Δp in the pressure of the liquid that is given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o},$$



5. Pascal's Principle

- <u>Pascal's Principle and the Hydraulic Lever</u>

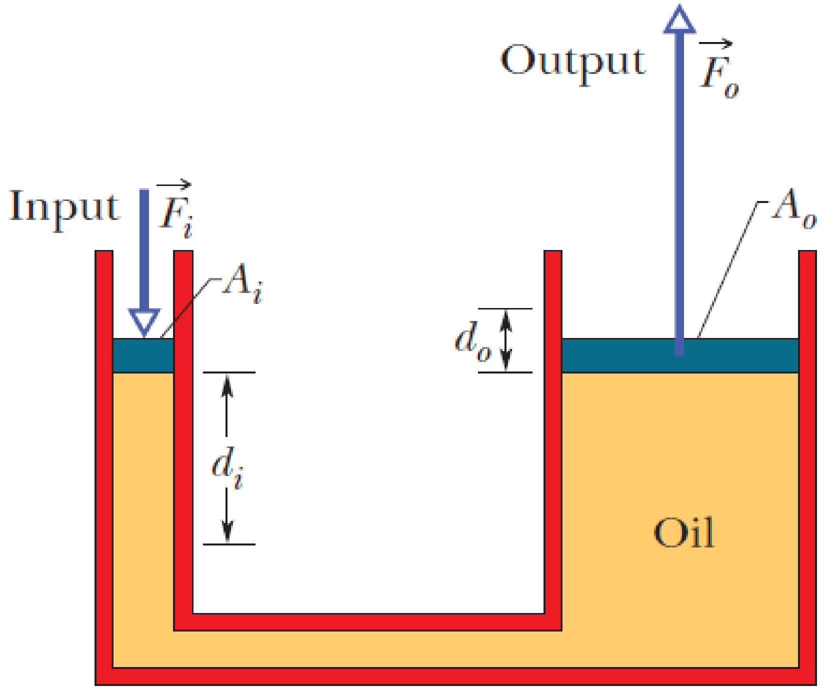
$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o},$$

so

$$F_o = \frac{A_o}{A_i} F_i.$$

In we move the input piston downward a distance d_i, the output piston moves upward a distance d_o, such that the same volume of the incompressible liquid is displaced at both pistons. Then

$$V = A_i d_i = A_o d_o,$$



5. Pascal's Principle

- <u>Pascal's Principle and the Hydraulic Lever</u>

$$V = A_i d_i = A_o d_o,$$

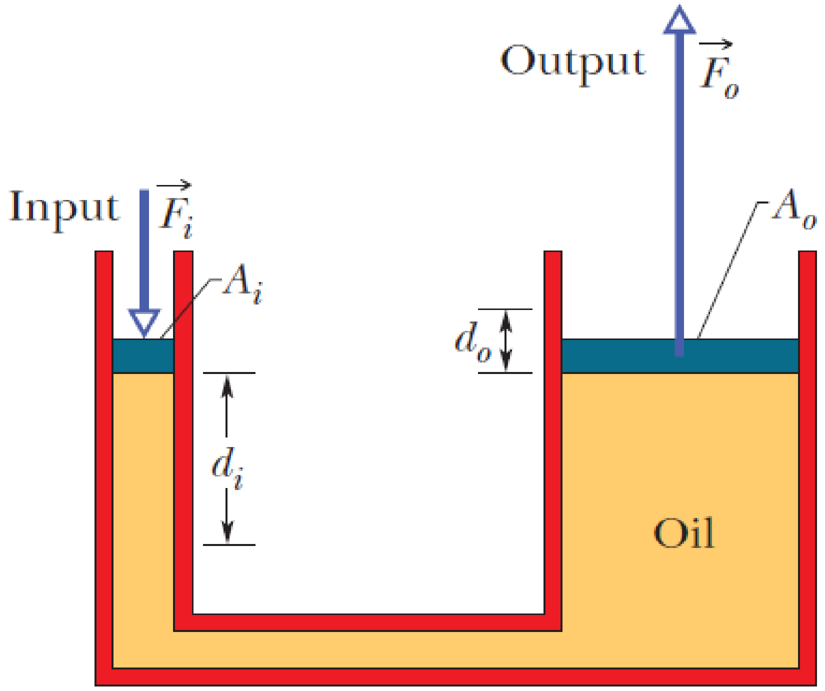
which we can write as

$$d_o = d_i \frac{A_i}{A_o}.$$

The output work is

$$W = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right) = F_i d_i.$$

The work W done on the input piston is equal to the work W done by the output piston in lifting the load.

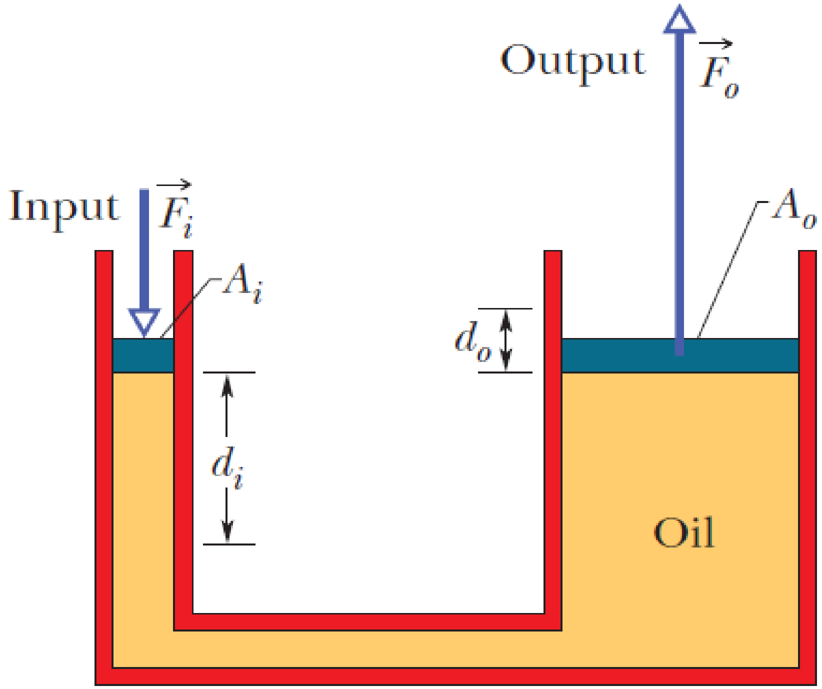


5. Pascal's Principle

- Pascal’s Principle and the Hydraulic Lever

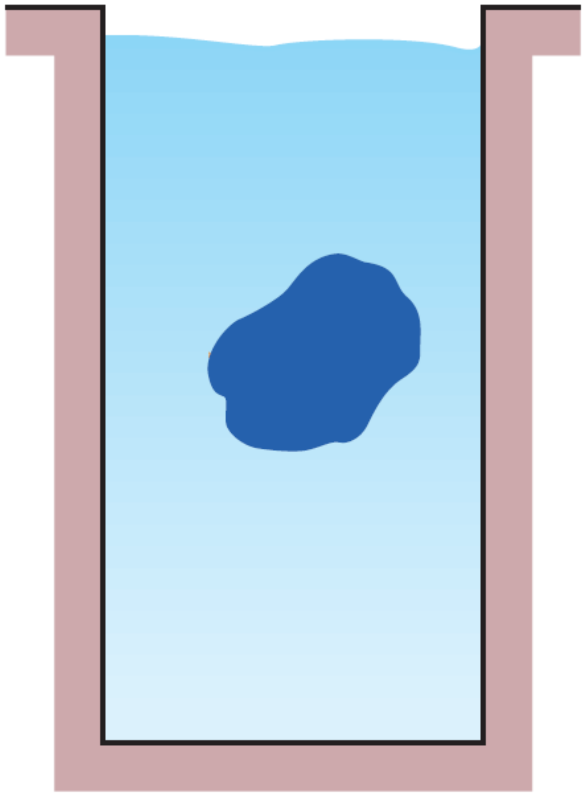
  The advantage of a hydraulic lever is:

  With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.



6. Archimedes' Principle

- Consider the situation in the figure where a sack of water is in static equilibrium. The sack experiences an upward force from the surrounding water, opposing its weight. This upward force is a **buoyant force** $\vec{F}_b$. It is due to the fact that the water near the bottom of the sack is greater than the pressure near its top.
- The magnitude of the buoyant force $\vec{F}_b$ is equal to the magnitude $m_f g$, where m_f is the mass of the fluid (water). Thus the magnitude of the buoyant force is equal to the weight of water in the sack.



6. Archimedes' Principle

- Archimedes' Principle: When a body is fully or partially submerged in a fluid, a buoyant force from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body.
- The buoyant force on a body in a fluid has the magnitude

$$F_b = m_f g,$$

where m_f is the mass of the fluid.

6. Archimedes' Principle

- Floating:

  When a lightweight object is placed in water, the object moves into the water because of the gravitational force. The magnitude of F_b increases as the object displaces more and more water. The object comes to rest when $F_b = F_g$. The object is then in static equilibrium and said to be floating.

  We can write

$$F_b = F_g,$$

  or

$$F_g = m_f g.$$

6. Archimedes' Principle

- <u>Apparent weight</u>:

  If we measure the weight of a stone, then repeat the measurement underwater we get a different reading. The upward buoyant force on the stone decreases the second reading. That second reading is an **apparent weight**. An apparent weight is related to the actual weight of a body and the buoyant force on the body by

$$\text{weight}_{\text{app}} = \text{weight} - F_b.$$

  What is the apparent weight of a floating body?

6. Archimedes' Principle

CHECKPOINT 2

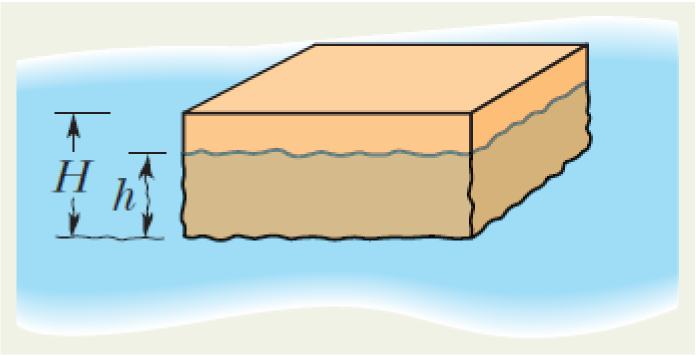
A penguin floats first in a fluid of density ρ_0, then in a fluid of density $0.95\rho_0$, and then in a fluid of density $1.1\rho_0$. (a) Rank the densities according to the magnitude of the buoyant force on the penguin, greatest first. (b) Rank the densities according to the amount of fluid displaced by the penguin, greatest first.

(a) All tie.

(b) $0.95\rho_0$, ρ_0, $1.1\rho_0$.

6. Archimedes' Principle

Example 4: In the figure, a block of density $\rho = 800 \text{ kg/m}^3$ floats face down in a fluid of density $\rho_f = 1200 \text{ kg/m}^3$. The block has height $H = 6.0 \text{ cm}$.



(a) By what depth h is the block submerged?

For a floating body

$$F_b = F_g.$$

We know that $F_b = m_f g = \rho_f V_f g = \rho_f A h g$. Also, $F_g = mg = \rho V g = \rho A H g$. We therefore can write

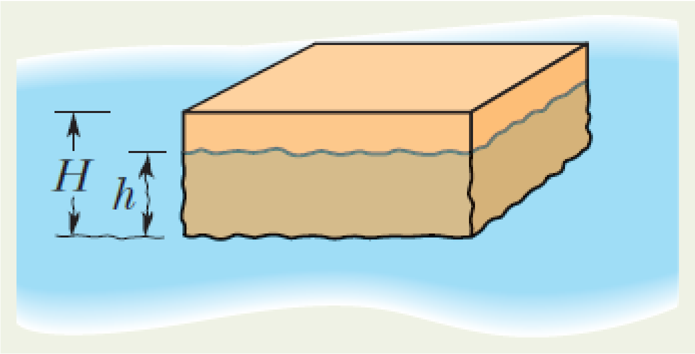
$$\rho_f h = \rho H.$$

Solving for h and substituting,

$$h = \frac{\rho}{\rho_f} H = \frac{800 \text{ kg/m}^3}{1200 \text{ kg/m}^3}(6.0 \text{ cm}) = 4.0 \text{ cm}.$$

6. Archimedes' Principle

(b) If the block is held fully submerged and then released, what is the magnitude of its acceleration?



Newton's second law for components along an axis normal to the block is

$$F_b - F_g = ma.$$

or

$$\rho_f AHg - \rho AHg = \rho AHa.$$

Solving for a and substituting yield

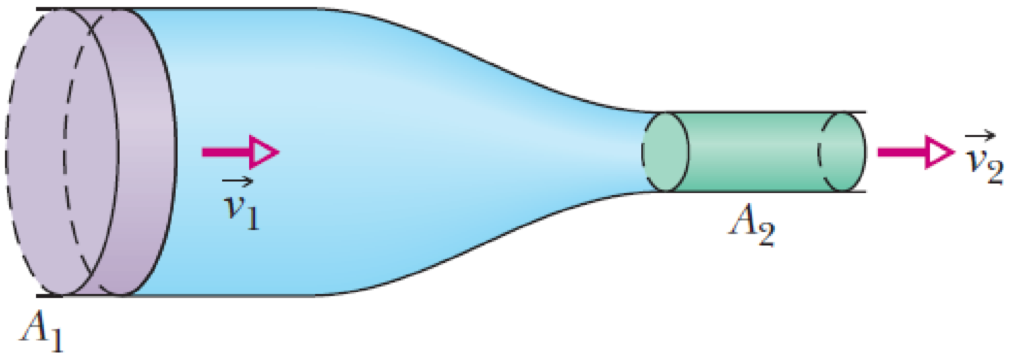
$$a = \left(\frac{\rho_f}{\rho} - 1\right) g = \left(\frac{1200\ \mathrm{kg/m^3}}{800\ \mathrm{kg/m^3}} - 1\right)\left(9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) = 4.9\ \frac{\mathrm{m}}{\mathrm{s}^2}.$$

7. The Equation of Continuity

- We want to relate v and A for the steady flow of an ideal fluid through a tube with varying cross section. During a time interval Δt a volume $\Delta V = A_1 v_1 \Delta t$ of a fluid enters the left end of the tube. An identical volume $\Delta V = A_2 v_2 \Delta t$ must emerge from the right end, since the fluid is incompressible. We therefore write

$$A_1 v_1 = A_2 v_2.$$

- This equation is called the **equation of continuity**.



7. The Equation of Continuity

- We can rewrite the continuity equation as

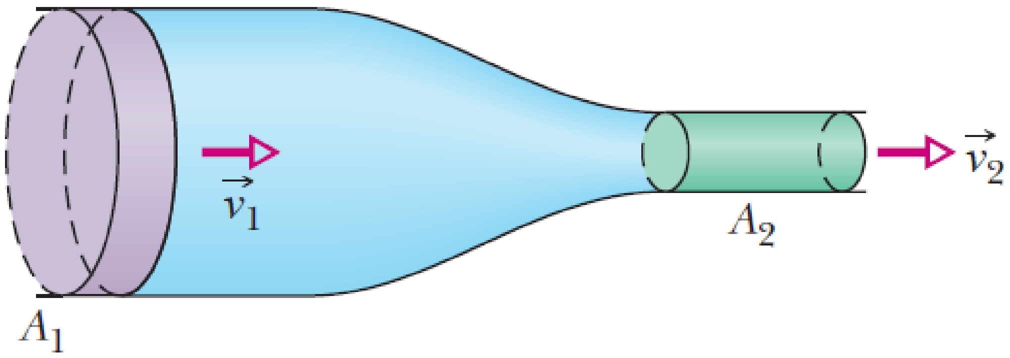
$$R_V = Av = \text{a constnat},$$

where R_V is the volume flow rate of the fluid. Its SI unit is m^3/s.

- If the density ρ of the fluid is uniform we can multiply the last relation by ρ to get the **mass flow rate** R_m:

$$R_m = \rho R_V = \rho A v = \text{a constnat}.$$

The SI unit of R_m is $\mathrm{kg/s}$.

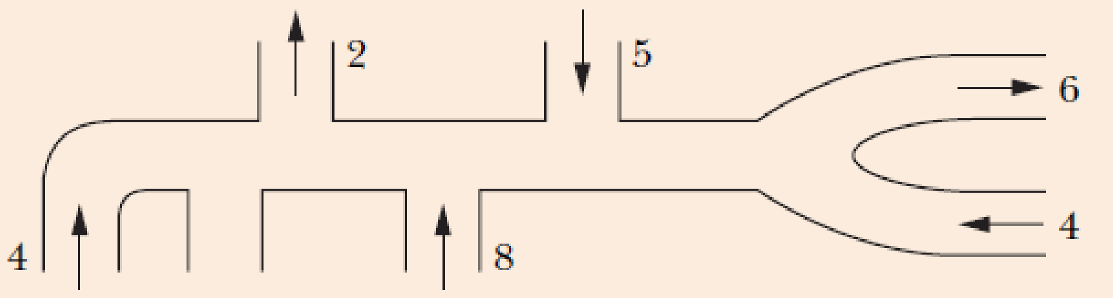


7. The Equation of Continuity

CHECKPOINT 3

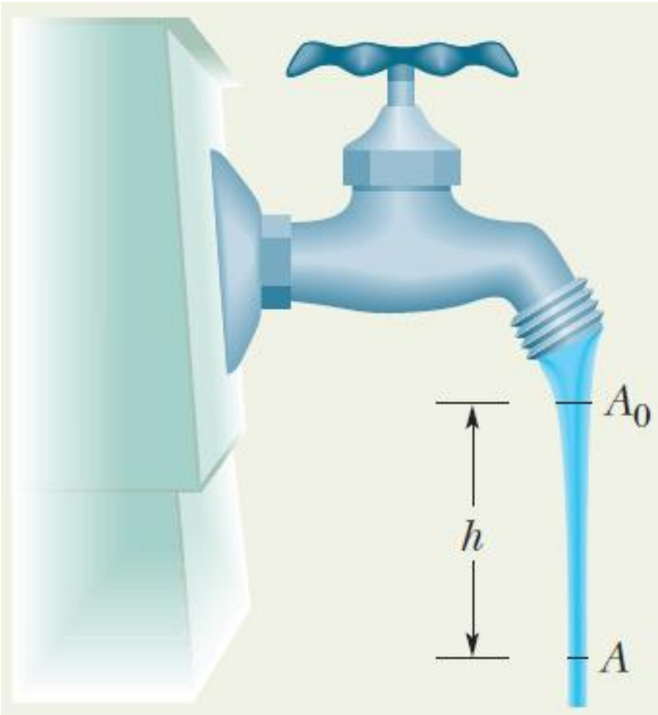
The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?

13 cm^3/s outward.



7. The Equation of Continuity

Example 5: The figure shows how the stream of water emerging from a faucet "necks down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (nonturbulant) falling stream because the gravitational force increases the speed of the stream. Here the indicated cross-sectional areas are $A_0 = 1.2\ \mathrm{cm}^2$ $A = 0.35\ \mathrm{cm}^2$. The two levels are separated by a vertical distance $h = 45\ \mathrm{mm}$. What is the volume flow rate from the tap?



The volume flow rate through the higher cross section must be the same as that through the lower cross section:

$$A_0 v_0 = A v.$$

7. The Equation of Continuity

$$A_0 v_0 = Av.$$

Because the water is falling freely

$$v^2 = v_0^2 + 2gh.$$

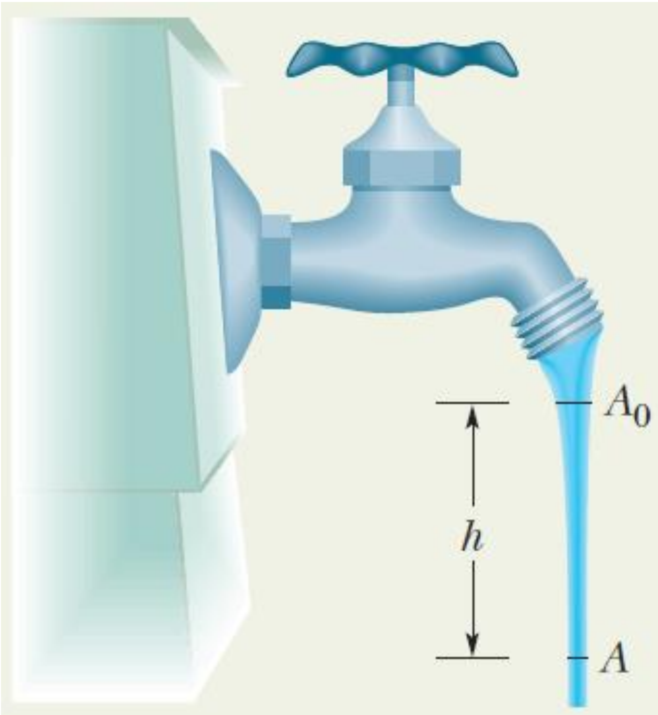
Substituting for v in the last equation and solving for v_0 give

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}} = \sqrt{\frac{2(9.8\ \mathrm{m/s^2})(0.045\ \mathrm{m})(0.35\ \mathrm{cm^2})^2}{(1.2\ \mathrm{cm^2})^2 - (0.35\ \mathrm{cm^2})^2}}$$

$$= 0.286\ \mathrm{m/s}.$$

The volume flow rate is then

$$R_V = A_0 v_0 = (1.2\ \mathrm{cm^2})(28.6\ \mathrm{cm/s}) = 34\ \mathrm{cm^3/s}.$$



8. Bernoulli's Equation

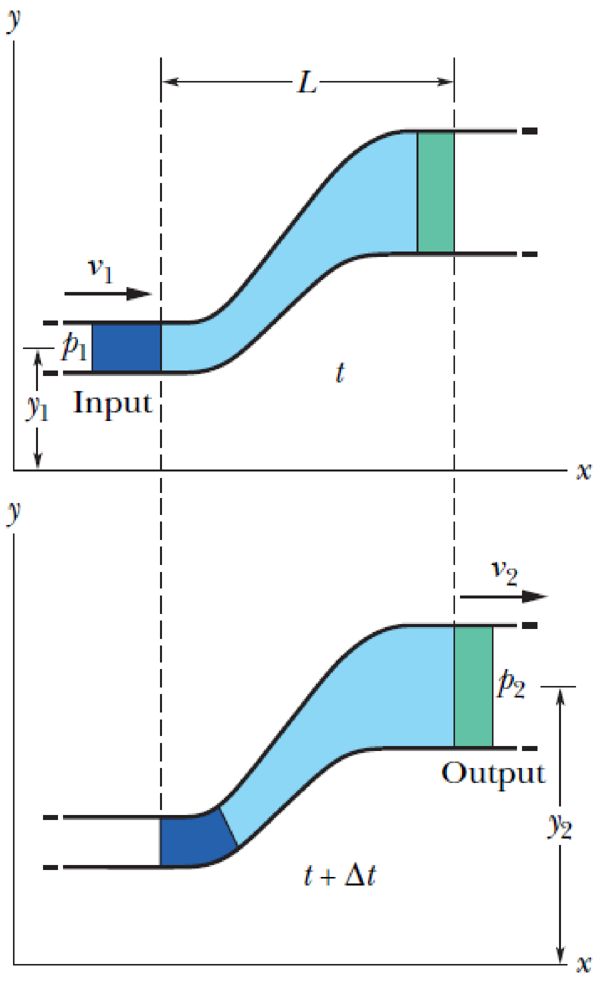
- Consider the situation in the figure where an ideal fluid is flowing through the tube at a steady rate. By applying the principle of conservation of energy to the fluid, we relate v, p and y at the ends of the tube by

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

- The term $\frac{1}{2}\rho v^2$ is called the fluid's **kinetic energy density**.
- We can also write

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant.}$$

- These are two equivalent forms of **Bernoulli's equation**.

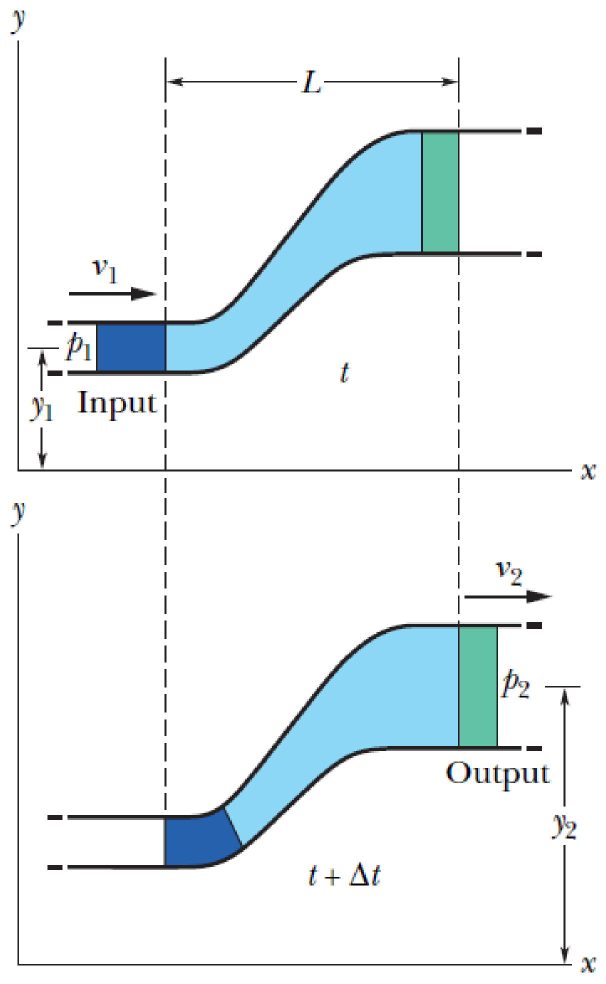


8. Bernoulli’s Equation

- When $y_1 = y_2$ the Bernoulli’s equation becomes

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2.$$

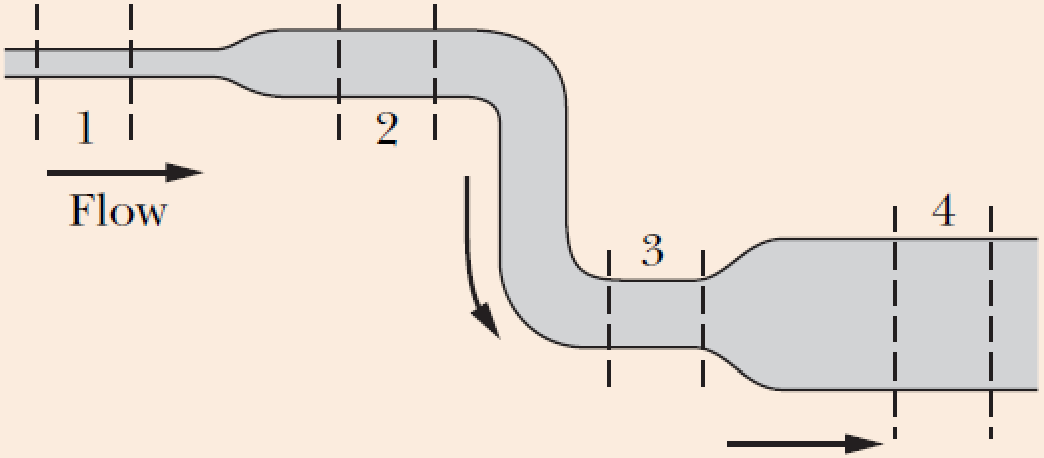
- This equation predicts that if the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.



8. Bernoulli's Equation

CHECKPOINT 4

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.



(a) All tie.

(b) 1, 2 & 3 tie, 4.

(c) 4, 3, 2, 1.

8. Bernoulli's Equation

Example 6: Ethanol of density $\rho = 791\,\mathrm{kg/m^3}$ flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.20 \times 10^{-3}\,\mathrm{m^2}$ to $A_2 = A_1/2$. The pressure difference between the wide and narrow sections of the pipe is $4120\,\mathrm{Pa}$. What is the volume flow rate R_V of the ethanol?

The continuity equation $A_1 v_1 = A_2 v_2$ tells us that

$$v_2 = \frac{A_1}{A_2} v_1 = 2v_1.$$

Using Bernoulli's equation $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$ with $y_1 = y_2$ and rearranging we get

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho(4v_1^2 - v_1^2) = \frac{3}{2}\rho v_1^2.$$

8. Bernoulli's Equation

Solving for v_1 yield

$$v_1 = \sqrt{\frac{2}{3\rho}(p_1 - p_2)}.$$

The volume flow rate is then

$$R_V = A_1 v_1 = A_1 \sqrt{\frac{2}{3\rho}(p_1 - p_2)}$$

$$= (1.20 \times 10^{-3}\ \mathrm{m}^2)\sqrt{\frac{2(4120\ \mathrm{Pa})}{3(791\ \mathrm{kg/m^3})}} = 2.23 \times 10^{-3}\ \mathrm{m^3/s}.$$

8. Bernoulli's Equation

Example 7: What is the speed v of the water exiting a tank though a hole a distance h below the water surface?

Bernoulli's equation for the problem reads

$$p_0 + \frac{1}{2}\rho v_0^2 + \rho g h = p_0 + \frac{1}{2}\rho v^2 + \rho g(0),$$

where v_0 is the speed of the water through the tank. Because the area of the base tank is much larger than the hole's area, v_0 is very small and we can neglect the term $\frac{1}{2}\rho v_0^2$. Solving for v we get

$$v = \sqrt{2gh}.$$