Let the absorption spectrum $\alpha(\omega)$ of a sample be centered at ω_0 and have a Lorentzian profile with a maximum value of one and a full-width at half maximum of γ. Also, let the length of the sample to be 0.01 so that $\alpha L \ll 1$. Use the approximation $e^{-\alpha} \approx 1 - \alpha L$.

Suppose a laser beam with frequency ω_{L0} and power P_0 is sent through the sample and the laser frequency is modulated sinusoidally such that the laser frequency becomes $\omega_L = \omega_{L0} + a \sin \Omega t$, where a is the modulation depth and Ω is the angular frequency of modulation. Suppose you are using an ideal lock-in amplifier to detect the transmitted power. Your lock-in amplifier multiplies the transmitted power by $\sin(n\Omega t + \phi)$, and take the average over on period of the modulation frequency and divides the result by the average of P_0 over one modulation period. Here n is a positive integer and ϕ is a phase constant that needs to be adjusted for each order n to get the maximum value and the right sign of the derivative. When modulating with $\sin \Omega t$ and assuming no phase is accumulated during measurement process, multiply by $\sin(1\Omega t + 0)$ to extract the first derivative, multiply by $\sin(2\Omega t - \pi/2)$ to extract the second derivative, and multiply by $\sin(2\Omega t - \pi)$ to extract the third derivative.

Q1.

Use Mathematica to simulate your ideal lock-in amplifier action and plot the output of the amplifier as a function of a dimensionless quantity $x = \frac{\omega - \omega_0}{\gamma}$ from $x = -2$ to $x = +2$ for the following cases

$a = 0.05\,\gamma$ and $n = 1$.

$a = 0.05\,\gamma$ and $n = 2$.

$a = 0.05\,\gamma$ and $n = 3$.

Q2.

Find expressions for the first, second and third derivatives of $\alpha(\omega)$ with respect to ω.

Use the last equation in page 10 of the 5th edition of Demtröder book “laser spectroscopy 2 Experimental Techniques” to find an approximate expression for the output of your lock-in amplifier for $n = 1, 2$ and 3. Use only the first term of each square bracket.

Q3.

Plot the expressions you obtain in Q2 as function of x from $x = -2$ to $x = +2$ for the same cases in Q1. Compare the approximate result of Q3 and the exact result of Q1 by overlapping the corresponding plots.

Q4.

Plot the exact and approximate result of your lock-in amplifier for the case $n = 1$ and modulation depth of 0.2.

Q5.

Plot the exact result of your lock-in amplifier for the case $n = 1$ and for different modulation depths and find the modulation depth that results in the maximum signal of the lock-in amplifier.