Q1.

At some location, the electric field is described by

$$E(t) = \begin{cases} 0, & t < 0 \\ E_0 \sin \omega_0 t \, e^{-(\gamma/2)t}, & t \geq 0. \end{cases}$$

Carry out the Fourier transform of this field and find an expression for its intensity spectrum $I(\omega)$. Do not do any approximation and assume that $I(\omega_0) = 1$. Please, show your work in detail.

Q2.

Show that for $\omega_0 \gg |\omega_0 - \omega|$ and $\gamma \ll \omega_0$, the intensity spectrum has a Lorentzian profile. What is the full-width at half maximum $\Delta\omega$ of this spectrum. Please, show your work in detail.

Q3.

Plot on the same plot the exact expression from Q1 and approximate expression from Q2 of the intensity spectrum for $\gamma = 0.2\ \omega_0$ in the range $-2\gamma < \omega - \omega_0 < +2\gamma$.

Plot on the same plot the exact expression from Q1 and approximate expression from Q2 of the intensity spectrum for $\gamma = 0.002\ \omega_0$ in the range $-2\gamma < \omega - \omega_0 < +2\gamma$.

What is a typical value for γ/ω_0 for an atomic transition in the visible region, where γ is the natural line width?

Q4.

The intensity $I(t) \propto |E(t)|^2 \propto e^{-\gamma t} = e^{-t/\tau}$. What is the relation between the full width at half maximum of the intensity spectrum $\Delta\omega$ and τ?