Assume that, in a laser, the upper energy level $|2\rangle$ and the lower energy level $|1\rangle$ have equal statistical weigh, the photon loss factor from the cavity is β, the gain medium is pumped to the upper energy level at rate of P, the relaxation rate of the upper energy level is R_2 (excluding spontaneous emission to the lower level), the relaxation rate of the lower level is R_1 and the spontaneous emission probability is A_{21}, the Einstein coefficient of induced emission is B_{21}, the laser operating at a frequency of ν with band width of $\Delta\nu$, the number density of the lower and upper level are N_1 and N_2, respectively, and the photon number density is n.

Q1.

Solve numerically the rate equations and plot the inversion population and the photon density as function of time from $t = 0$ to 10 s for the following values:

$\beta = 10\ s^{-1}$

$A_{21} = 2\ s^{-1}$

$P = 100\ s^{-1}$

$R_1 = 2.5\ s^{-1}$

$R_2 = 1\ s^{-1}$

$B_{21} h\nu\ \Delta\nu = 2\ cm^3\ s^{-1}$

$N_1(0) = 10\ cm^{-3}$

$N_2(0) = 0\ cm^{-3}$

$n(0) = 1\ cm^{-3}$

Q2.

Use the rate equations to derive an expression for the pump rate at threshold. Calculate the value of the pump at threshold for the values in Q1.

Q3.

For $P = 50, 100, 150, 200, and\ 250\ s^{-1}$, find numerically the steady state value for the inversion population and photon number density. calculate the threshold population inversion ΔN_{thr} and compare it with the steady state values of the population inversion you find numerically for pumping rate above threshold.

Q4.

Plot the steady state values of photon number density as a function of pumping rate for pumping rate above threshold. Check that the line connecting the photon number density intercepts the pumping rate axis at the threshold pumping rate.