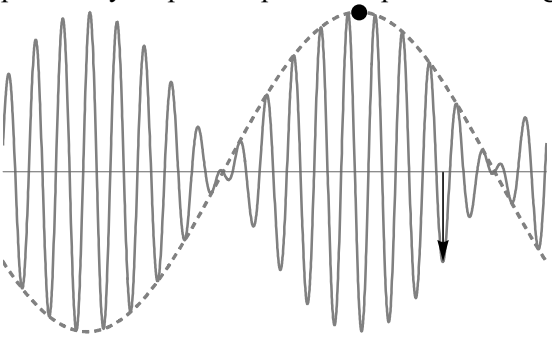
A- Two sinusoidal waves with the displacements $y_1(x,t) = A\cos(k_1x - \omega_1 t)$, and $y_2(x,t) = A\cos(k_2x - \omega_2 t)$ interfere to produce a wave with displacement $y(x,t) = y_1(x,t) + y_2(x,t)$. Use some trigonometric identities to write $y(x,t)$ as product of two traveling waves.

B- Suppose that $k_2 - k_1 \ll k_1 \approx k_2$ and $\omega_2 - \omega_1 \ll \omega_1 \approx \omega_2$, show that the speed of one of the waves in the product is $v_p \approx \omega_2/k_2 \approx \omega_1/k_1$, while the speed of the other wave in the product is $v_g = d\omega/dk = (\omega_2 - \omega_1)/(k_2 - k_1)$.

C- Use Mathematica to generate three animated GIF files as follows:

	File1	File2	File3
k_1	$1 + 0.05$	$1 + 0.05$	$1 + 0.05$
k_2	$1 - 0.05$	$1 - 0.05$	$1 - 0.05$
ω_1	$1 - 0.025$	1	$1 + 0.025$
ω_2	$1 + 0.025$	1	$1 - 0.025$

You need to choose the options in your plots to produce a plot like the figure below:



The disk should move at speed of v_g.
The arrow should move at speed of v_p.
Both the disk and arrow should start at $t = 0$ at $x = 20\pi$.
$A = 1$.
Range of x is from 0 to 40π.
Range of y is from -2.1 to 2.1.
Make frames for $t = 0$ to 20π in steps of $0.01\times 20\pi$.
The size of the disk in y direction is 0.1.
Adjust the size of the disk in the x direction to get a perfect circle.
The colors of the envelope and the wave are gray. The envelope should be dashed.
Show only the x-axis with no ticks.