The infinite sum

$$V(x,y)=\frac{4}{\pi}\sum_{n=1,3,5\ldots}\frac{1}{n}e^{-\frac{n\pi x}{a}}\sin\left(\frac{n\pi y}{a}\right)$$

can be summed exactly to the following function

$$V(x,y)=\frac{2}{\pi}\tan^{-1}\left(\frac{\sin\left(\frac{\pi y}{a}\right)}{\sinh\left(\frac{\pi x}{a}\right)}\right).$$

You will use Mathematica to see how a limited number of terms in the sum approaches the exact function. You will do two cases:

Case 1:

Produce five different plots for $x=0$ and $-0.1\le y/a\le 1.1$. Use the following option in your plots: $Range->\ \{0,1.3\}$.

- Plot of the exact function and just one term of the sum.
- Plot of the exact function and three terms of the sum.
- Plot of the exact function and 10 terms of the sum.
- Plot of the exact function and 100 terms of the sum.
- Plot of the exact function and 1000 terms of the sum.

Note since $\sinh 0=0$, Mathematica gives the following error when you try to plot the exact function $V(0,y)$: Power::infy: "Infinite expression 1/0 encountered.". To overcome this problem, use $Limit[V[x,y],x->0]$, instead.

Case 2:

Produce four different plots for $x=0.02\,a$ and $-0.1\le y/a\le 1.1$. Use the following option in your plots: $Range->\ \{0,1.3\}$.

- Plot of the exact function and just one term of the sum.
- Plot of the exact function and three terms of the sum.
- Plot of the exact function and 10 terms of the sum.
- Plot of the exact function and 20 terms of the sum.

Note you do not need to use “Limit” in this case.

What is your observation with respect to the number of terms needed for the sum to approach the exact function for the two cases?