Lab 05: Carnot Cycle

Objective

To plot a p-V diagram and T-S diagram for a Carnot cycle using Excel.

Introduction

In this lab, you will use Excel to draw a p-V diagram and T-S diagram for a Carnot cycle. You will also create an animation for the change of the state of the gas and energies for a Carnot engine and Carnot refrigerator.

A heat engine is a device that converts some of the energy it absorbs as heat from a high-temperature reservoir into work and expels the rest of the heat to a low-temperature reservoir. Let us call the temperature of the high-temperature reservoir T_H and the temperature of the low-temperature reservoir T_L . Any engine has a working substance, such as a gasoline-air mixture in a car engine, which must be taken over thermodynamic cycles to produce work on a sustained basis.

A Carnot engine is an ideal heat engine that uses an ideal gas as its working substance. All processes in a Carnot engine are reversible. Compared to any heat engine operating between T_H and T_L , a Carnot engine is the most efficient one.

p-V diagram of a Carnot Engine

We want to plot the p-V diagram for the cycle of a Carnot engine which consists of four strokes as shown in the Fig. 1. In the first stroke, the temperature is kept constant at T_H and the volume of the ideal gas expands from V_a to V_b . The process in which the temperature is kept constant is called an isothermal process. In the second stroke, no heat is transferred to the gas and the gas expands from V_b to V_c and its temperature decreases from T_H to T_L . The process in which no heat is transferred to the gas is called an adiabatic process. In the third stroke, the gas is compressed isothermally at T_L and its volume decreases from V_c to V_d . In the fourth stroke, the gas is compressed adiabatically to its initial state at the beginning of the cycle.



Figure 1. a p-V diagram of a Carnot cycle.

For a reversible adiabatic process:

 $TV^{\gamma-1} = \text{constant.}$ (1)

Here, $\gamma \equiv C_p/C_V$, where C_p is the molar specific heat of an ideal gas at constant pressure and C_V is the molar specific heat of an ideal gas at constant volume. $C_p = C_V + R$. We will use a monatomic gas, for which $C_V = 3R/2$ and $\gamma = 5/3$.

For the first isothermal stroke

$$T = T_H.$$
 (2)

For the second adiabatic stroke

$$TV^{\gamma-1} = T_H V_b^{\gamma-1} \to T = T_H (V_b / V)^{\gamma-1}.$$
 (3)

For the third isothermal stroke

$$T = T_L. (4)$$

For the fourth adiabatic stroke

$$TV^{\gamma-1} = T_H V_a^{\gamma-1} \to T = T_H (V_a/V)^{\gamma-1}.$$
 (5)

For any stroke, the pressure
$$p$$
 as a function of volume V can be found from the ideal gas law:
 $pV = nRT$. (6)
Here, n is the number of moles of the gas, and $R = 8.31$ J/mol·K is the gas constant.

Table 1 shows the volume at beginning of each stroke.

Quantity	Formula
n	Given
T_H	Given
T_L	Given
Va	Given
V_b	Given
V _c	$T_H V_b^{\gamma-1} = T_L V_c^{\gamma-1} \to V_c = V_b \left(\frac{T_H}{T_L}\right)^{1/(\gamma-1)}$
V _d	$T_H V_a^{\gamma - 1} = T_L V_d^{\gamma - 1} \to V_d = V_a \left(\frac{T_H}{T_L}\right)^{1/(\gamma - 1)}$

Table 1. Volume at the beginning of each stroke.

Exercise 1:

You will use Excel to plot p-V diagram of a cycle of a Carnot engine.

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10 Vc m3 Volume at c 11 Vd m3 Volume at d 2 Image: Comparison of the second		в	/ D	E F	G	Н	
13	1	R	J/mol K	gas constant			
15	2	Cv	J/mol K	molar specific heat at	t constant ve	olume	
16	3	Ср	J/mol K	molar specific heat at	constant p	ressure	
18	4	γ		gamma = Cp/Cv			
20	5	n	mole	number of moles		1.000	
21 4-Select Page sert Draw Page Layout	6	TH	K	temperature of high-	temperatur	e reservoir	
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24 main menu, then Margins Orientation Size Prir	8	Va	m3	Volume at a			
25 select · · · Area	9	Vb	m3	Volume at b			
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28 select Landscape	11	Vd	m3	Volume at d			
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32 8.51 J/mork ga							
33							
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clicking on its name and	change it to L	ab-05					
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1- To change the name of cell C1 to Rg, select cell C1, then click on the name box and change the text from	A B C	fx		6- chang A and B location	je the widths by double cl of the arrowl	of column icking at the heads.
can use Rg to refer to the content of cell C1.	R Cv	J/m J/m	vpe the text.	c	D	E
		32				
		33	Str	oke Volume	e Pressure	Temper.
Name cell C2 CV.	0.000	34		V (m3)) p (Pa)	т (К)
Name cell C4 gamma or increase	1.xisx Search	35	а			
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Name cell C6 TH digits after		7- In	cell C35, type	=Va.	. .	
Name cell C7 TL. the decimal	General		ot the path on th	ne p-V diag	gram for stro	ke $a \rightarrow b$,
Name cell C8 Va. point to	ter ~ \$ ~ % 🦻 號 🔆	you volume	will find the pres	sure value	tor 20 differ	ent
Name cell C9 Vb. match the			nes equally spa		en v _a and v _i	p.
Name cell C10 Vc. ones shown.	Number		ii Coo, type –C	50+(vb-va	1)/20.	
Name cell C11 Vd.			AB	C	DE	
			32 32 Churches Ma			
3-			33 Stroke Vo	(m 2)	(Da) T (V	er.
In cell C1, type 8.31.	B	D	34 V	(m3) p	(Pa) I (K	. <u>) </u>
In cell C2, type =3*Rg/2.	1 R 8.31	J/mol K	35 a	0.100		
In cell C3, type =Cv+Rg.	2 Cv 12.5	J/mol K	30	0.108		
In cell C4, type =Cp/Cv.	3 Cp 20.8	J/mol K Copy	the formula in	C36 to the	cells bellow	till cell C55
In cell C5, type 0.001.	4 γ 1.67	by cl	, ickina on the ria	ht-bottom	corner of ce	II C36.
In cell C6, type 600.	5 n 0.0010	mole holdi	ng. and moving	the mouse	e down to ce	Il C55 then
In cell C7, type 300.	6 TH 600	K relea	asing the mouse	. You shou	uld get the v	alue of Vb
In cell C8, type 0.100.	7 TL 300	K at ce	ell C55.	20.04		
In cell C9, type =2.5*Va.	8 Va 0.100	m ³ In ce	ell A55, type b.	A	B C	
In cell C10, type =Vb*(TH/TL)^(1/(gamma-1)).	9 Vb 0.250	m3		54	0.	.243
In cell C11, type =Va*(TH/TL)^(1/(gamma-1))	10 Vc 0.707	m3		55 b	0.	.250
	11 Vd 0.283	m3		EC		

1- To plot the path on the p-V diagram for stroke $b \rightarrow c$
you will find the pressure value for 20 different volumes
equally spaced between Vb and Vc.
In cell C56, type =C55+(Vc-Vb)/20.
Copy the formula in cell C56 to the cells bellow till cell
C75. You should get the value of Vc at cell C75.
In cell A75, type c.

2- To plot the path on the p-V diagram for stroke $c \rightarrow d$, you will find the pressure value for 20 different volumes equally spaced between Vc and Vd. In cell C76, type =C75+(Vd-Vc)/20. Copy the formula in cell C76 to the cells bellow till cell

C95. You should get the value of Vd at cell C95.

In cell A95, type d.

3- To plot the path on the p-V diagram for stroke $d \rightarrow a$, you will find the pressure value for 20 different volumes equally spaced between Vd and Va. In cell C96, type =C95+(Va-Vd)/20. Copy the formula in cell C96 to the cells bellow till cell C155. You should get the value of Va at cell C155. In cell A115, type a.

4-

In cell B35, type 1. Copy it to the cells below it till cell B54 by clicking on the right-bottom corner of cell B35, holding, and moving the mouse down to cell B54 then releasing the mouse. In cell B55, type 2 and copy it to the cells below it till cell B74. In cell B75, type 3 and copy it to the cells below it till cell B94. In cell B95, type 4 and copy it to the cells below it till cell B115.

In cell E76, type
Copy the formula

cell E95.

95	300	
In cell E96, type =	TH*(Va/C96	i)^(gamma-1).
Copy the formula in	cell E96 to t	the cells below it till
cell E115.		

300

300

J.	E
113	536
114	566
115	600

1	A	В	С
94			0.304
95	d		0.283

B

A

74

75 c

A

114

115 a

B

C

C

0.109

0.100

0.684

0.707

In cell E35, typ Copy the form cell E55.	oe = ⁻ ula in (TH. cell E35 to	o the cells below it till
	-	-	
	54	600	
	55	600	
	EC		

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In cell E56, type =TH*(Vb/C56)^(gamma-1) Copy the formula in cell E56 to the cells below it till cell E75.

1	E
73	314
74	307
75	300
70	

=TL

93

94

a in cell E76 to the cells below it till Ε



 1- To change the decimal places, double click on any number of the horizontal axis to show the Format Axis menu on the right of the worksheet. In the bottom of the Format 	3- Click on Edit. Change the series name to Carnot Cycle. Then press OK.	Legend Entries (Series)
Axis menu, open Number tab and change the Decimal places to 1.	Click on Add.	Legend Entries (Series)
Change the decimal places for the vertical axis to 0. 2- move the mouse over the curve and right-click on it. From the menu choose Select Data.	Type a for the Series name. Select cell C35 for the Series X values. Select cell D35 for the Series Y values. Then press OK.	t Series ? × ies name: f = a ies X values: ab-05'!\$C\$35 $f = 0.100ies Y values:ab-05'!$ \$D\$35 $f = 49.9OK Cancel$
10 Delete 0 0.0 0.0 0.2 Vol Change Series Chart Type Select Data 3-D Rotation Add Data Labels	 Add another series with the name b a and select cell D55 for the Y values. Add another series with the name c at and select cell D75 for the Y values. Add another series with the name d a and select cell D95 for the Y values. 	nd select cell C55 for the x values nd select cell C75 for the x values nd select cell C95 for the x values Then press OK.

1- Double click on your plot to show the Format menu on the right of the worksheet. Click on the arrow on the right of Chart Options and select Series "Carnot Cycle" as shown.

Change the color of the Carnot Cycle curve to gray as shown.





Gas state of a Carnot engine

The state of an ideal gas is determined by its volume, pressure, and temperature. If the number of moles does not change, as in the case of a Carnot engine, then the state of the gas is determined only by its pressure and volume. For a Carnot engine, the state of the gas changes in a clockwise manner over the Carnot cycle. We will assume the cycle starts at state a of Fig. 1.

Exercise 2:

You will use Excel to make animation for the state of the ideal gas in the p-V diagram as it changes over the cycle of a Carnot engine. You will use a red dot to indicate the current state of the gas.







T-S diagram of a Carnot engine

The entropy of a system S is a state property that does not depend on path. For an ideal gas, the change in entropy ΔS is

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}$$

You will plot temperature T as a function of change of entropy with respect of state $a: \Delta S = S - S_a$ for a cycle of a Carnot engine.

Exercise 3:

You will use Excel to plot a T-S diagram a cycle of a Carnot engine.

1- Type the text. E F Chagne in 33 Temper. Entory 34 T (K) (S-Sa) (mJ/K)

Note the unit is **milli** Joule per Kelvin (mJ/K).

Adjust the width of F column by double clicking on the line between letter F and letter G in the column titles.

2-

In cell F35, type =1000*(n*Rg*LN(C35/Va)+n*Cv*LN(E35/TH)).

Note we multiply by 1000 because our unit is in milli.

Copy the formula in F35 to the cells bellow till cell F115 by clicking on the right-bottom corner of cell F35, holding, and moving the mouse down to cell F115 then releasing the mouse.

In cell F29, type =F34. In cell F30, type =INDIRECT("F"&rn).

14	F	G
29	(S-Sa) (mJ/K)	
30	5.00	
31		
32		
	Chagne in	
33	Entory	
34	(S-Sa) (mJ/K)	
35	0.00	
36	0.60	
37	1.16	

3- Right click on the chart area but outside the plot area and select copy as shown.

Carnot Cycle	Fill Outline
	👗 Cu <u>t</u>
b	Basta Ontions:
0.4 0 Volume (m ³)	Right click here yle
-0	A Eont

<u>Select any empty cell</u> and then press Ctrl key and v key at the same time to paste the copied chart.

Move the pasted chart just to the right of the original chart.

Change the axis title for the horizontal axis to Change in Entropy (S-Sa) (mJ/K).

Select the letter a of Sa, then press Ctrl key and 1 key at the same time. Then select subscript.

Change the axis title for the vertical axis to Temperature T (K).

1- Right-click on the past select from the menu Sel Select Series Carnot Cyc then select Edit.	ed chart and ect Data Legend Entries (Series)	 2- As you did for the P-V diagram: Change the color of series Carnot Cycle to gray. For series a, b, c, and d, select built-in marker circle, make
	✓ candid Cycle ✓ a ✓ b ✓ c ✓ d	its size 3, and make its color orange. For Series state, select built-in marker circle, make its size 5, and make its color red.
	Edit Series 😓 ? Series <u>n</u> ame:	Add data label for series a, b, c, and d. For a and b make them above the data point, while for c and d make them below the data point.
In the X values change letter C with F. In the Y values change letter D with E.	="Carnot Cycle" 1 = Carnot Series <u>X</u> values: = 'lab-05'!\$C\$35:\$C\$115 1 = 0.100, 0 Series <u>Y</u> values: = 'lab-05'!\$D\$35:\$D\$115 1 = 49.9, 46 OK Ca	 3- Change the range for the horizontal axis from 0 to 8 and for the vertical axis from 200 to 700. Make Units Major 100 for the vertical axis. Format Axis Axis Options Text Options Text Options
Do the same for Series a Series b Series d Series c Series state		Axis Options Bounds Minimum 200.0 Reset Maximum 700.0 Reset Units Major 100.0 Reset



Work and heat for a Carnot engine

Work W and heat Q are not state properties; they depend on path. In the following, you will find the accumulated work and heat as the gas state follows a Carnot cycle starting at state a of Fig. 1 and just for one cycle. You will get different values if you start with another state or go over more than one cycle. The internal energy E_{int} of a system is a state property that does not depend on path, and the change in the internal energy ΔE_{int} , according to the first law of thermodynamics, is

$$\Delta E_{int} = Q - W. \tag{7}$$

For an ideal gas $E_{int} = nC_V T$.

For an isothermal process, for which the temperature is constant,

$$T = \text{constant}$$

$$E_{int} = nC_V T = \text{constant}$$

$$Q = W = nRT \ln \frac{V_f}{V_i}$$
isothermal process.

For an adiabatic process, for which no heat is transferred to the gas Q = 0,

$$\begin{array}{l} \Delta E_{int} = n C_V \Delta T \\ Q = 0 \\ W = -n C_V \Delta T \end{array} \right\} \quad \text{adiabatic process.}$$

The table below shows how to calculate the accumulated work starting at state a for one cycle of a Carnot engine. The heat starting from state a can be obtained from the first law of thremodynamcis, Eq. 7.

Table 2. Work starting at point *a* to a state on the cycle of a Carnot engine for the first cycle.

On stroke	Work done by the gas in the first cycle <u>starting from state a</u>
$a \rightarrow b$	$nRT_H \ln \frac{V}{V_a}$
$b \rightarrow c$	$W_{a o b, Eng} - nC_V(T - T_H)$
$c \rightarrow d$	$W_{a \to c, Eng} + nRT_L \ln \frac{V}{V_c}$
$d \rightarrow a$	$W_{a \to d, Eng} - nC_V(T - T_L)$

$$W_{a \to b, Eng} = nRT_H \ln \frac{V_b}{V_a}$$
$$W_{a \to c, Eng} = W_{a \to b, Eng} - nC_V(T_L - T_H)$$
$$W_{a \to d, Eng} = W_{a \to c, Eng} + nRT_L \ln \frac{V_d}{V_c}$$

Exercise 4:

You will use Excel to calculate the internal energy E_{int} at any state of the Carnot cycle. Also, you will calculate the work W done the gas and heat Q absorbed by the gas starting at point a for the first cycle. You will use a chart plot the show ΔE_{int} , W, and Q as the state of the gas changes over the cycle of a Carnot engine.

1- Select cell H31 to K31 and merge them together. In the merged cell, type Starting at point a for one cycle. Type the text.

1	G	н	1	J	K	
31		Star	ting at poi	nt a for one	cycle	
32		Engine	Engine	Refriger.	Refriger.	
33	Internal Energy	Work	Heat	Work	Heat	
34 35	Eint (J)	(L) W	Q (J)	(L) W	Q (J)	

2-

Name cell G35	Einta
Name cell H55	WabEng.
Name cell H75	WacEng.
Name cell H95	WadEng.

3- In cell G35, type =n*CV*E35 Change the decimal places to 2 for cell G35. Copy the formula in G35 to the cells bellow till cell G115.

4-

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In cell H35, type =n^{R}g^{T}H^{L}N(C35/Va)
Copy the formula in H35 to the cells bellow till cell H55.
Change the decimal places to 2 for the range H35 to H55.
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In cell H56, type =WabEng-n*CV*(E56-TH) Copy the formula in H56 to the cells bellow till cell H75.

In cell H76, type =WacEng+n*Rg*TL*LN(C76/Vc) Copy the formula in H76 to the cells bellow till cell H95.

In cell H96, type =WadEng-n*CV*(E96-TL) Copy the formula in H96 to the cells bellow till cell H115.

4-

In cell I35, type =(G35-Einta)+H35 Change the decimal places to 2 for cell I35.

Copy the formula in I35 to the cells bellow till cell I115.



In cell I28, type =INDIRECT("G"&rn)-Einta. Change the decimal places to 2 for cell I28. In cell I29, type =INDIRECT("H"&rn). Change the decimal places to 2 for cell I29. In cell I30, type =INDIRECT("I"&rn). Change the decimal places to 2 for cell I30.

7- Select the range as shown

1	G	H	1
27			
28		ΔEint (J)	-0.792
29		(L) W	5.36
30		Q (J)	4.57
21		Ctarti	ng at naint a

1- Select Insert from the main menu, then from the chart group select Insert Column or Bar Chart as shown.

Home Insert Dr	aw Page	Lay	
ole Recommended PivotTables Tables	Pictures	j SI ∂ Ic ∂ SI	
Recommended	2-D Col	v ¶¶.v umn	6
Charts	ha		AF

Change Chart Title to Energy Change the range for the vertical axis to -10 to 10 and its decimal places to 0.





Press F9 key repeatedly and observe how $\Delta Eint$, W, and Q change over the cycle of a Carnot engine .

Carnot Refrigerator

A refrigerator is a device that uses work to transfer heat from a low-temperature reservoir to a high-temperature reservoir. A Carnot refrigerator is an ideal refrigerator that uses an ideal gas as its working substance. The gas state follows a cycle of a Carnot engine but in counterclockwise manner.

The table below shows you how to calculate the accumulated work starting at state a for one cycle of a Carnot refrigerator.

Table 3.	3. Work starting at point a to a state on the cycle of a Carnot refrigerator for the first				
	On stroke	Work done by the gas in the first cycle starting from state <i>a</i>			
	$a \rightarrow d$	$-nC_V(T-T_H)$			
	$d \rightarrow c$	$W_{a \to d, Ref} + nRT_L \ln \frac{V}{V_d}$			
	$c \rightarrow b$	$W_{a \to c, Ref} - nC_V(T - T_L)$			
	b ightarrow a	$W_{a \to b, Ref} + nRT_H \ln \frac{V}{V_b}$			

$$W_{a \to d, Ref} = -nC_V(T_L - T_H)$$
$$W_{a \to c, Ref} = W_{a \to d, Ref} + nRT_L \ln \frac{V_c}{V_d}$$
$$W_{a \to b, Ref} = W_{a \to c, Ref} - nC_V(T_H - T_L)$$

Exercise 5:

You will use Excel to calculate the work W done by the gas and heat Q absorbed by the gas starting at point a for the first cycle of a Carnot refrigerator. Also, you will modify your Excel worksheet so that you can choose an engine or refrigerator in the animations you did previously.

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1- Name cell J55 WabRef. Name cell J75 WacRef. Name cell J95 WadRef.	2- In cell J35, type =Wabb Copy the formula in J35 t Change the decimal place In cell J55, type =Wacb Copy the formula in J55 t In cell J75, type =Wadb Copy the formula in J75 t In cell J95, type =-n*Cv Copy the formula in J95 t	Ref+n*Rg*TH*LN(C35/Vb) o the cells bellow till cell J54. es to 2 for the range J35 to J54. Ref-n*Cv*(E55-TL) o the cells bellow till cell J74. Ref+n*Rg*TL*LN(C75/Vd) o the cells bellow till cell J94. /*(E95-TH) o the cells bellow till cell J115.	3- In cell K35, type =(G35-Einta)+J35 Change the decimal places to 2 for cell K35. Copy the formula in K35 to the cells bellow till cell K115.			
 4- You will use the text in cell K28 to indicate whether to operate the cycle as an engine or refrigerator. If the text is "eng" then it is an engine and if it is "ref" then it is a refrigerator. Name cell K28 sel In cell K28, type eng Change the color of cell K28 to red. 						
5- Cell D28 is the row number Change it from =IF(rn<35,35,IF(rn>114,35,rn To =SWITCH(sel,"ref",IF(rn<35 Value to switch Value to match1	8- Change the text in K28 to ref and press F9					
6- Change cell I29 from =INDIRECT("H"&rn) To =IF(sel="ref",INDIRECT("J"&rn),INDIRECT("H"&rn)).7- Change cell I30 from =INDIRECT("I"&rn) To =INDIRECT("I"&rn) To =IF(sel="ref",INDIRECT("K"&rn),INDIRECT("I"&rn)).				key repeatedly and observe P-V diagram, T- S diagram, and energy bar chart.		