

## Objectives

### Ch 17: Waves 2

17.1 Speed of sound

17.2 Traveling sound waves

17.3 Interference

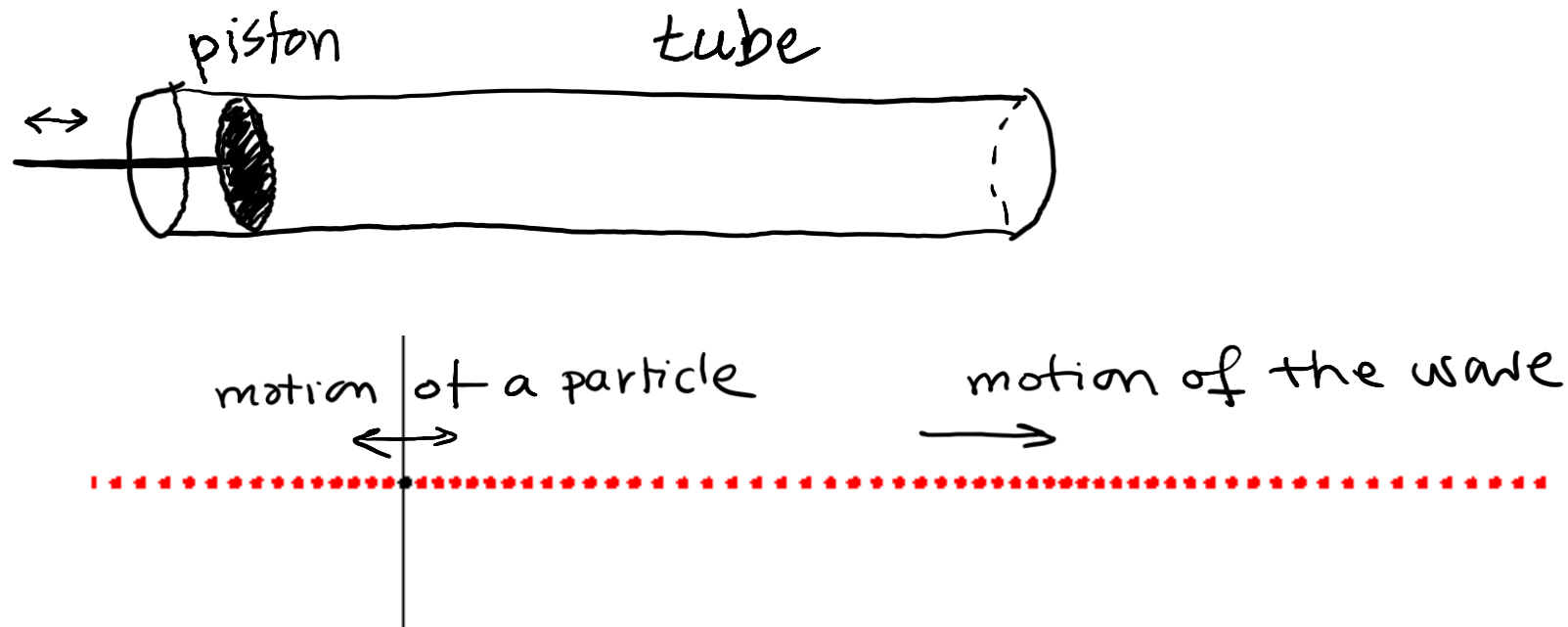
17.4 Intensity and sound level

17.5 ~~Sound waves resonances~~

17.7 ~~The Doppler effect~~

Sound waves are mechanical waves  $\Rightarrow$  They need medium to travel

Sound waves are longitudinal waves



## 17.1 Speed of Sound

$$v = \sqrt{\frac{B}{\rho}}$$

speed of sound  $\uparrow$   $v$

Bulk modulus  $\leftarrow B$

density of the medium  $\leftarrow \rho$

|            | $v \text{ (m/s)}$ |
|------------|-------------------|
| air (0°C)  | 331               |
| air (20°C) | 343               |
| water      | 1500              |
| steel      | 6000              |

$$B = - \frac{\Delta p}{\Delta V / V}$$

change in pressure  $\leftarrow \Delta p$

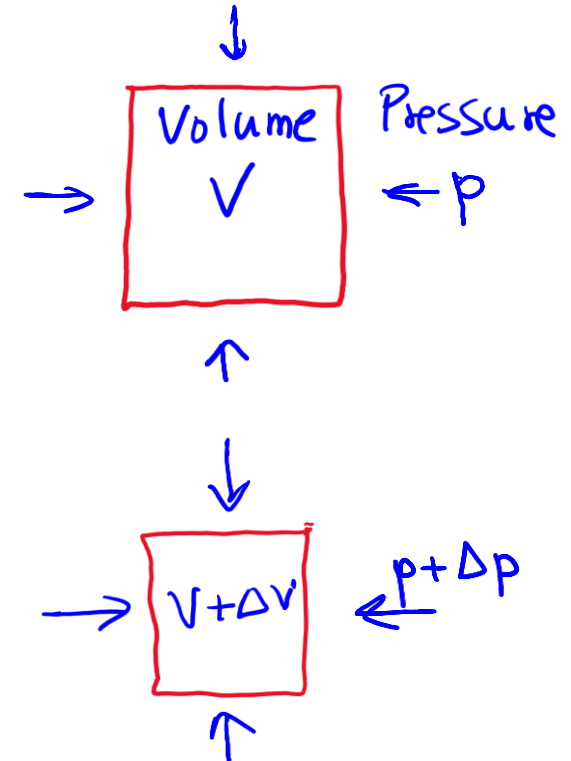
change in volume  $\leftarrow \Delta V$

original volume  $\leftarrow V$

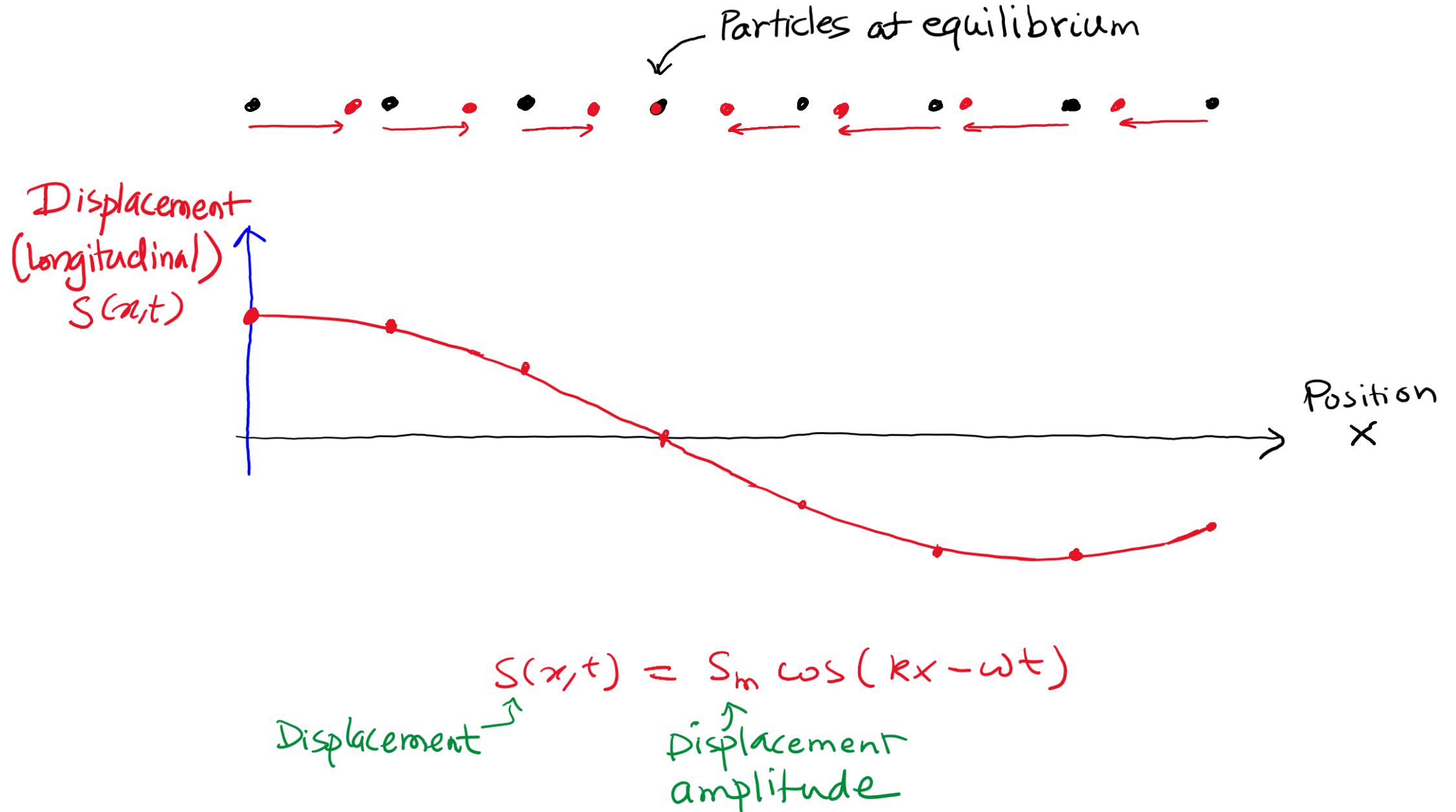
$B$  is always positive

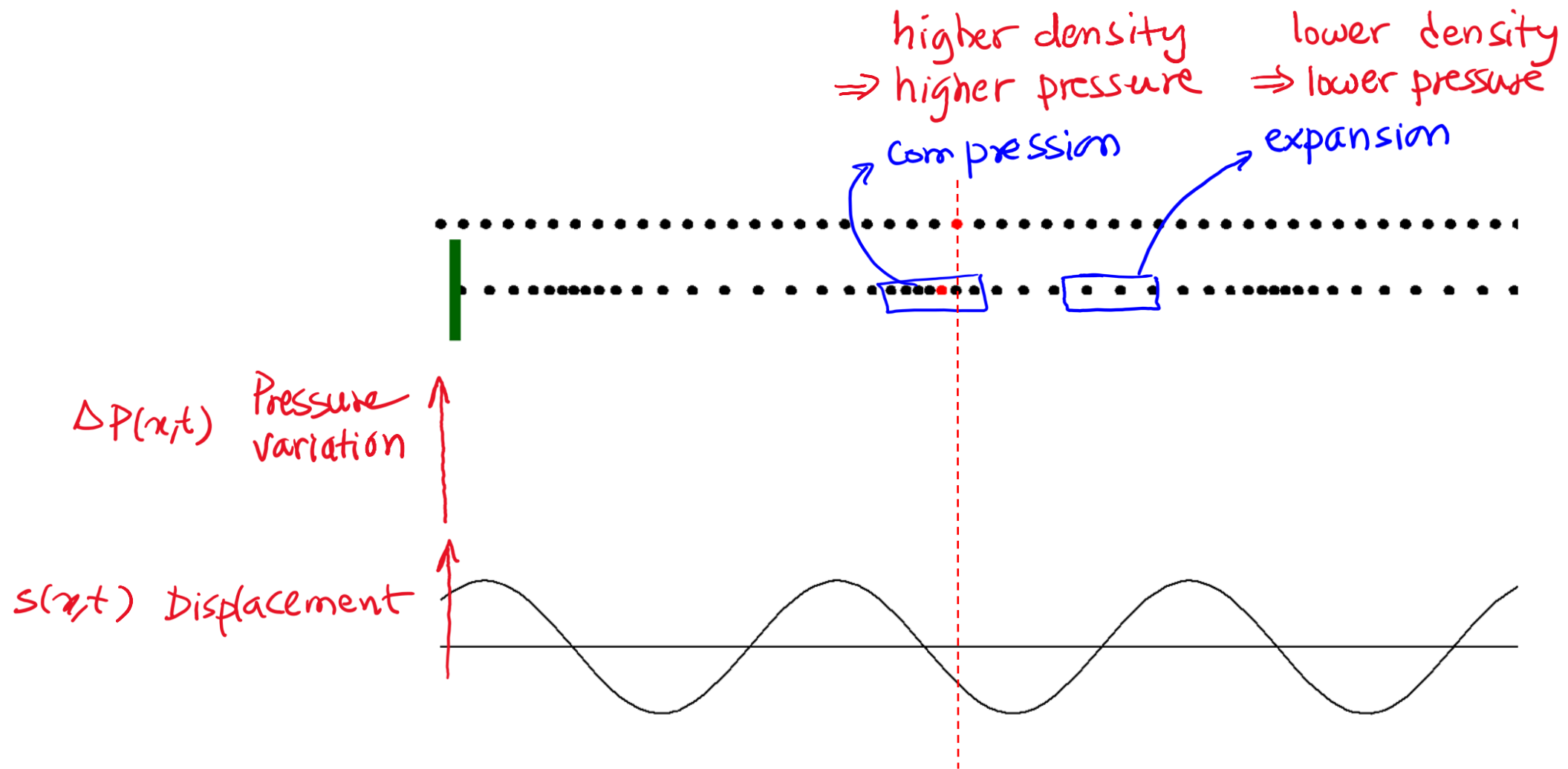
Higher  $B \Rightarrow$  less compressible medium

$$B_{\text{steel}} \gg B_{\text{air}}$$



## 17-2 Traveling sound waves





Displacement wave

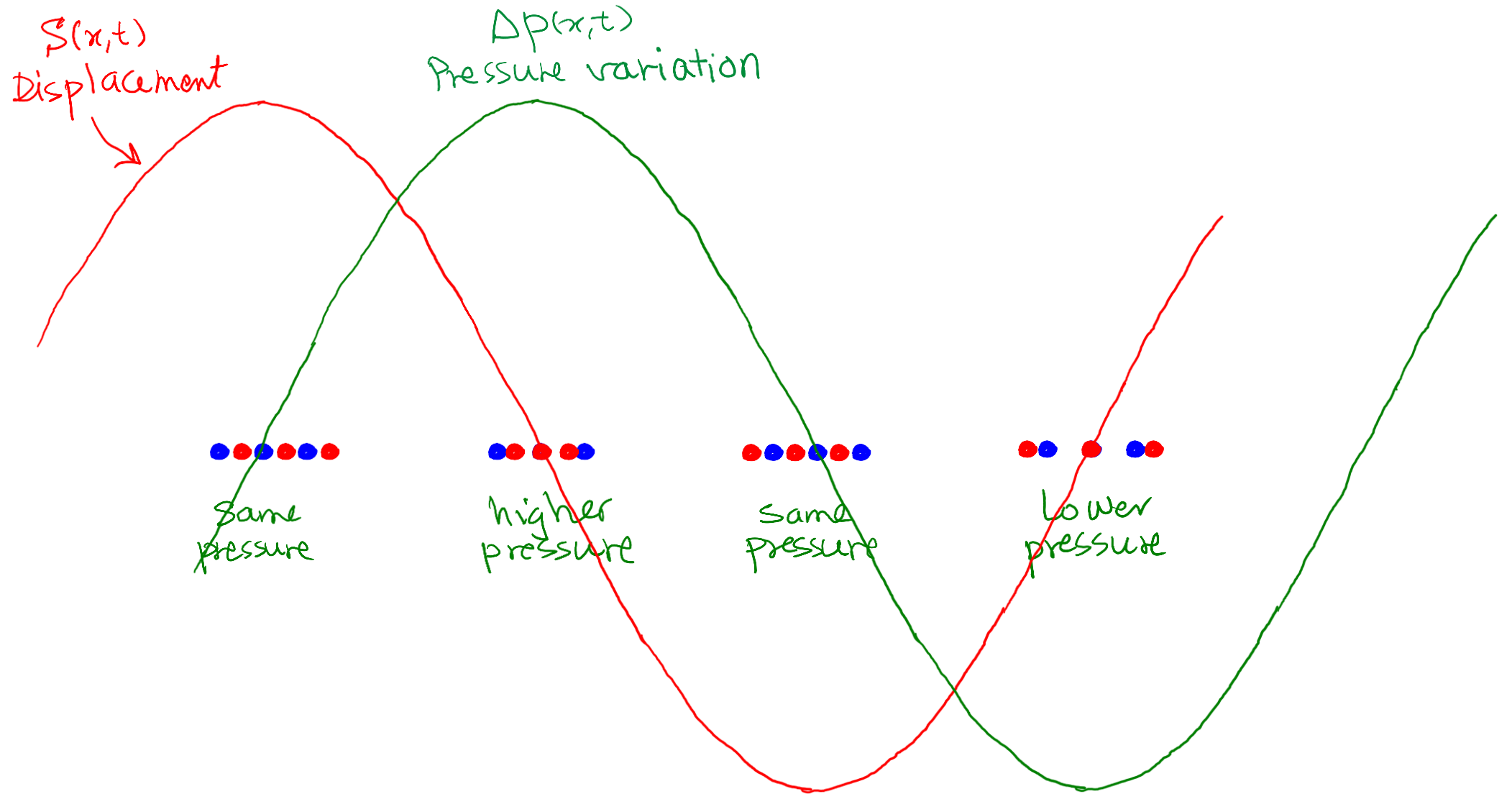
$$s(x,t) = s_m \cos(kx - \omega t)$$

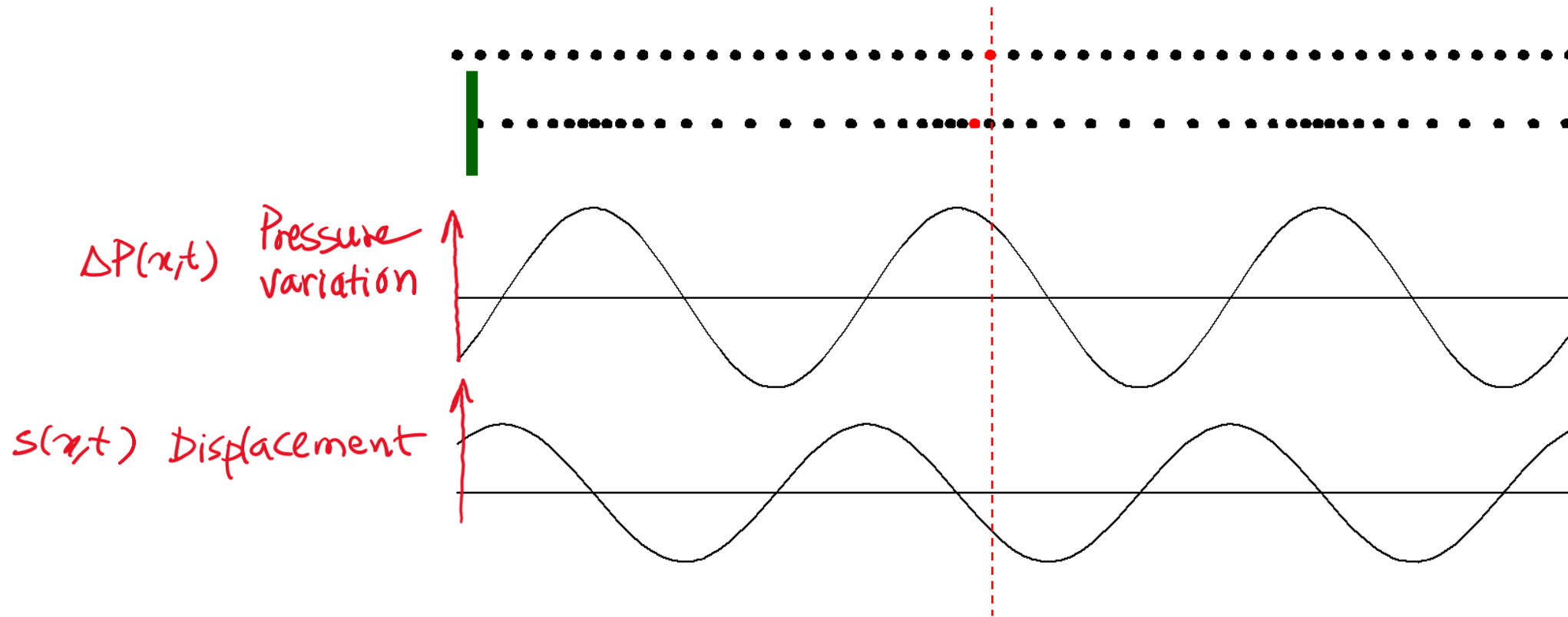
Displacement amplitude

Pressure wave

$$\Delta P(x,t) = \Delta p_m \sin(kx - \omega t)$$

Pressure amplitude





Displacement wave

$$s(x,t) = s_m \cos(kx - \omega t)$$

Displacement amplitude

Pressure wave

$$\Delta P(x,t) = \Delta P_m \sin(kx - \omega t)$$

Pressure amplitude

$$\Delta P_m = v \rho \omega S_m$$

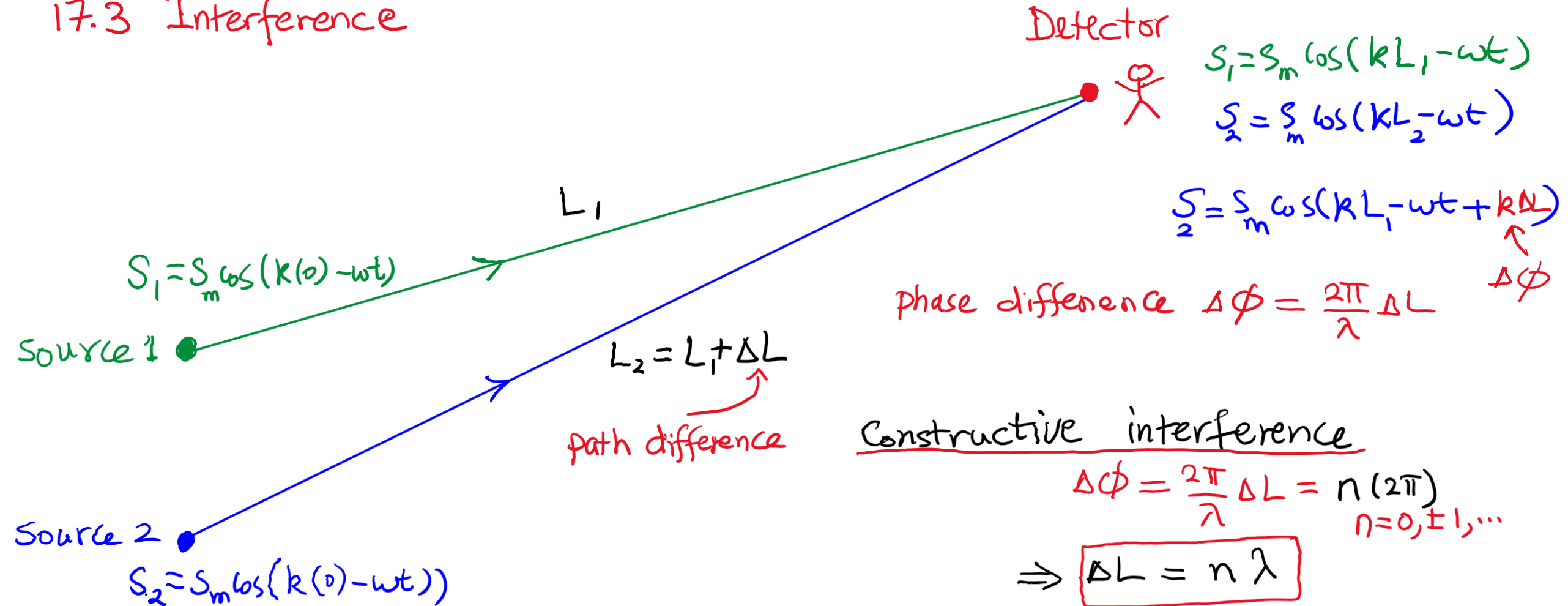
speed of sound  $\rightarrow v$   
 density  $\rightarrow \rho$   
 angular frequency  $\rightarrow \omega$   
 Displacement amplitude  $\rightarrow S_m$   
 Pressure amplitude  $\rightarrow \Delta P_m$

You can hear from  $f = 20 \text{ Hz}$  to  $20 \text{ kHz}$

Most sensitive at  $1000 \text{ Hz}$

| At $1000 \text{ Hz}$ | $S_m$                                  | $\Delta P_m$                             |
|----------------------|----------------------------------------|------------------------------------------|
| Loudest sound        | $10 \mu\text{m}$ $\leftarrow 10^{-6}$  | $30 \text{ Pa}$                          |
| Faintest sound       | $10 \text{ p m}$ $\leftarrow 10^{-12}$ | $30 \mu\text{Pa}$                        |
|                      | $\lambda = 0.34 \text{ m}$             | atmospheric Pressure = $10^5 \text{ Pa}$ |

## 17.3 Interference



Constructive interference

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L = n(2\pi) \quad n=0, \pm 1, \dots$$

$$\Rightarrow \boxed{\Delta L = n\lambda}$$

Destructive interference

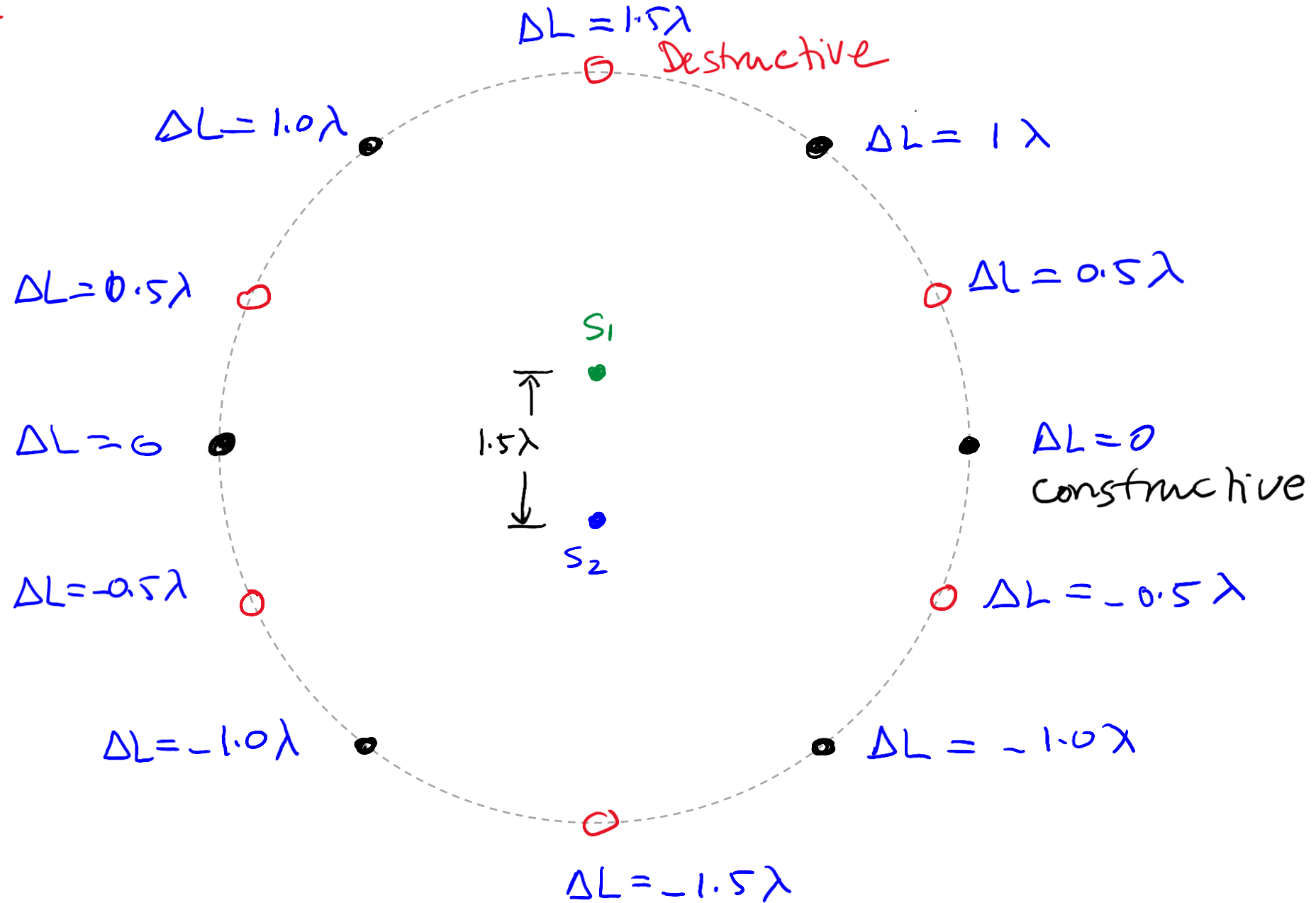
$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L = (2n+1)\pi$$

$$\Rightarrow \boxed{\Delta L = (n + \frac{1}{2})\lambda}$$

Two point sources emit sound waves of the same wavelength.

They are in phase.

Example



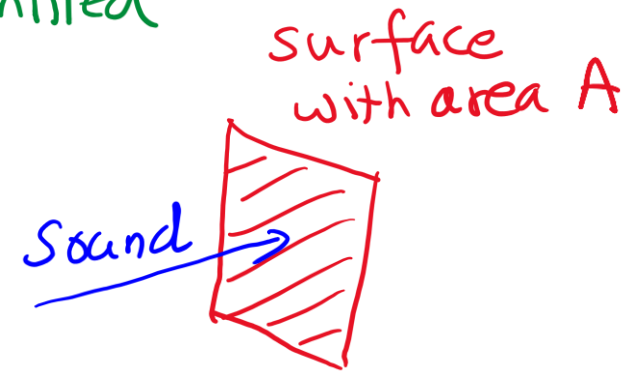
## 17.4 Intensity

Intensity of a sound wave

$$I \equiv \frac{P}{A}$$

Average power transmitted

area



For a sinusoidal wave

$$I = \frac{1}{2} \rho v \omega^2 S_m^2$$

density

speed of sound

angular frequency

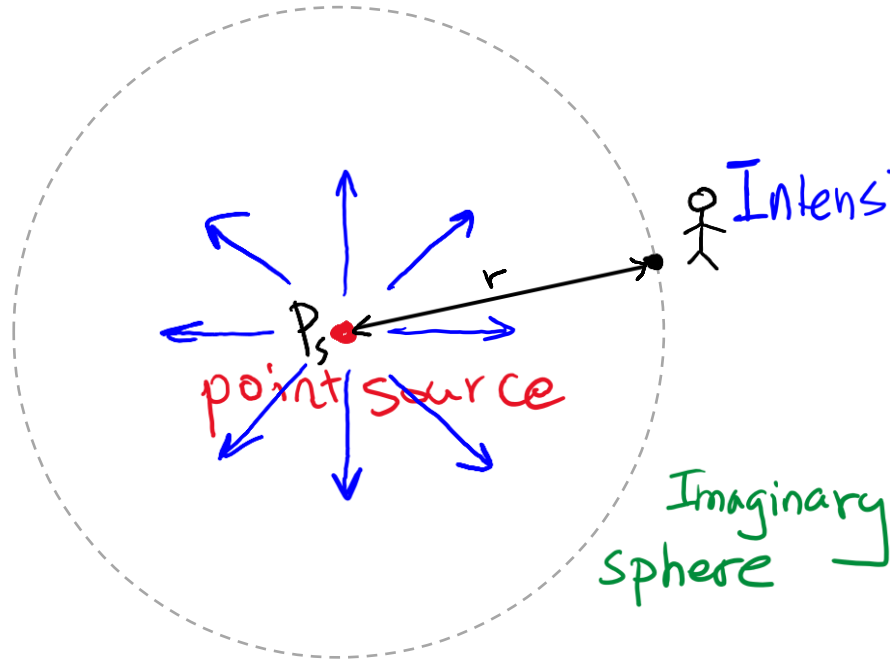
Displacement amplitude

Wave on a string

$$P_{\text{avg}} = \frac{1}{2} \mu v \omega^2 y_m^2$$

## Intensity at distance $r$ from a point source

For a point source, sound is emitted **isotropically** (equal intensity in all directions)



$P_s$  = power of the source

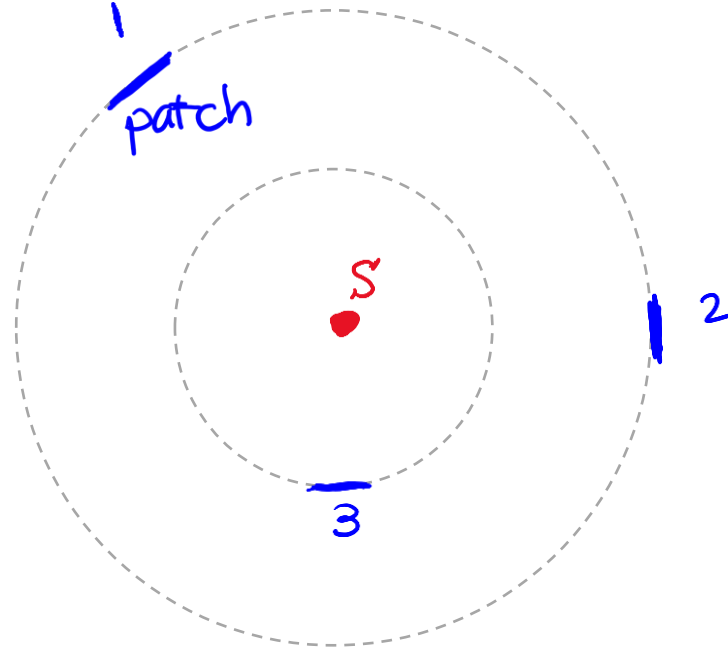
All energy of the source must pass through the surface of the sphere

The intensity at distance  $r$  from the source

$$I = \frac{P}{A} = \frac{\text{Power of the source}}{\text{Area of the sphere}} = \frac{P_s}{4\pi r^2}$$

Intensity decreases with the square of the distance from the source.

## Question



compare the intensity of sound at the patches

$$I = \frac{P_s}{4\pi r^2} \Rightarrow I_3 > I_1 = I_2$$

If the patches receive the same power, compare their areas

$$I = \frac{P}{A} \Rightarrow A = \frac{P}{I} \Rightarrow A_3 < A_1 = A_2$$

## Sound level

highest sound you can hear

$$S_m = 10 \mu\text{m} \Rightarrow I = 1 \frac{\text{W}}{\text{m}^2} \Rightarrow \beta = 120$$

lowest sound you can hear

$$S_m = 10 \text{ pm} \Rightarrow I = 10^{-12} \frac{\text{W}}{\text{m}^2} \Rightarrow \beta = 0$$

For a sinusoidal wave

$$I = \frac{1}{2} \rho v \omega^2 S_m^2$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$v = 343 \text{ m/s}$$

Most Sensitive  $f = 1000 \text{ Hz}$

$$\omega = 2\pi(1000) \frac{\text{rad}}{\text{s}}$$

Sound level  $\beta$  is a simpler way to express intensity

Sound level

$$\beta \equiv 10 \log \frac{I}{I_0}$$

Intensity

reference  
intensity  $\equiv 10^{-12} \frac{\text{W}}{\text{m}^2}$

measured in dB

decibel  
related to 10  
Graham Bell

## Example

An ear plug decreases the sound level by 20 dB.

$$\frac{\text{final intensity}}{\text{initial intensity}} = \frac{I_f}{I_i} = ?$$

$$\beta = 10 \log \frac{I}{I_0}$$

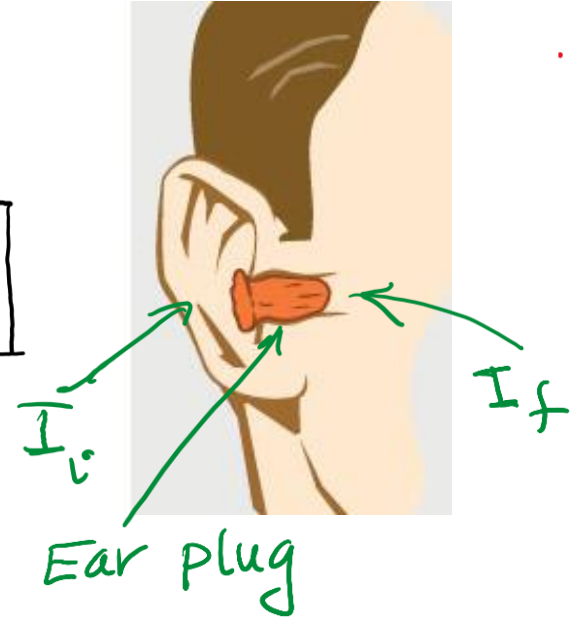
$$\beta_f - \beta_i = -20$$

$$10 \log \frac{I_f}{I_0} - 10 \log \frac{I_i}{I_0} = -20$$

$$\log \frac{I_f}{I_0} - \log \frac{I_i}{I_0} = -2 \Rightarrow \log \frac{I_f}{I_i} = -2$$

$$10^{\log \frac{I_f}{I_i}} = 10^{-2} \Rightarrow \frac{I_f}{I_i} = 10^{-2} = 0.01$$

$$\log a - \log b = \log \frac{a}{b}$$

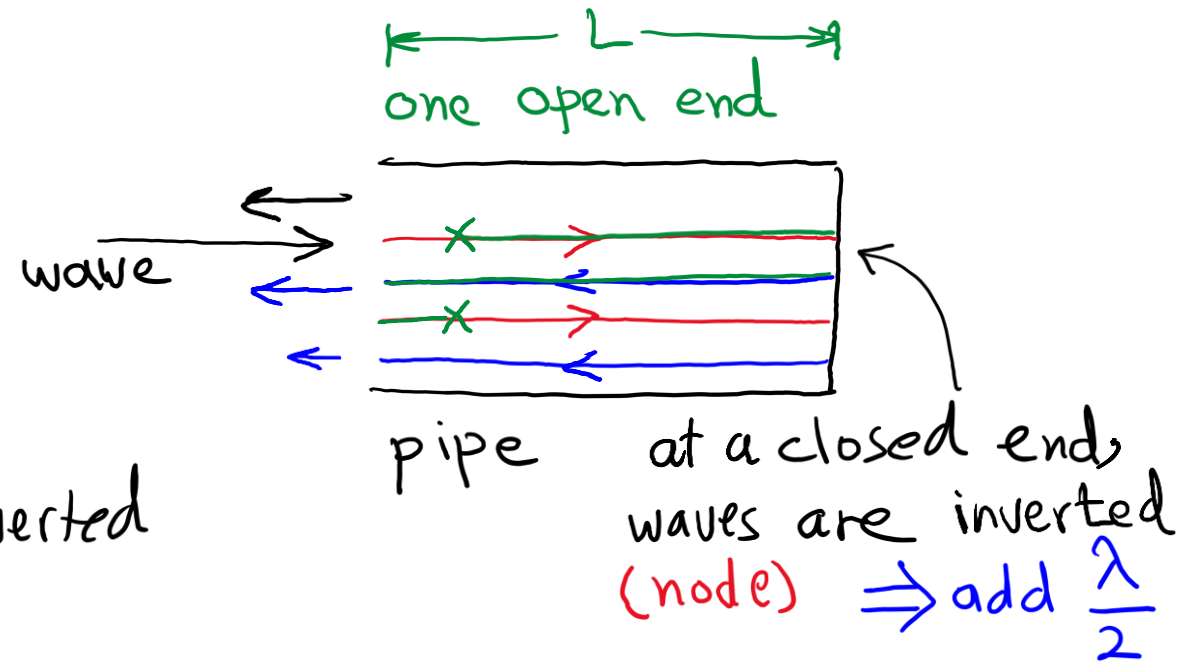
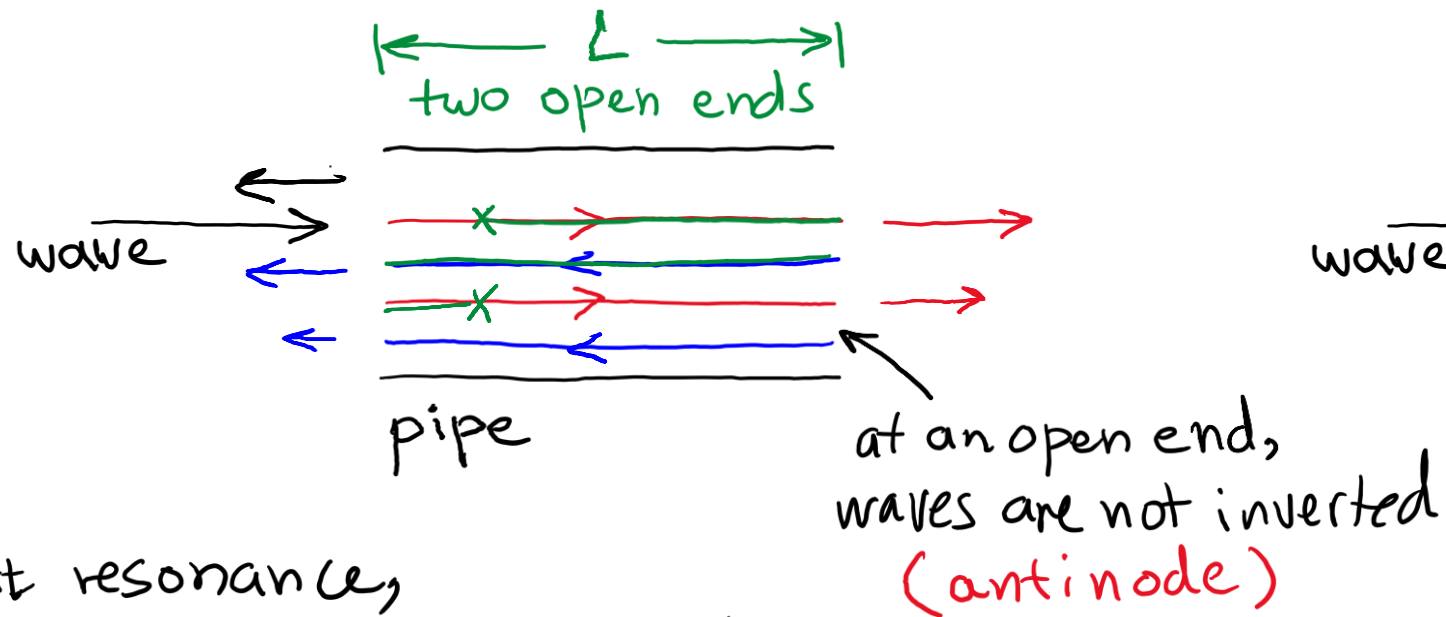


$\beta$  increases by 10 every time

the sound intensity is multiplied by 10

$$\Delta \beta = 10x \Rightarrow \frac{I_f}{I_i} = 10^x$$

## 17-5 Sound Wave resonance



At resonance,  
right-going waves are in phase

$$2L = n\lambda$$

$\nwarrow$   
 $n = 1, 2, 3, \dots$

$$2L + \frac{1}{2}\lambda = \bar{n}\lambda$$

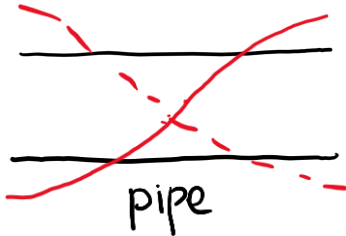
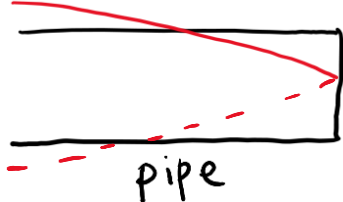
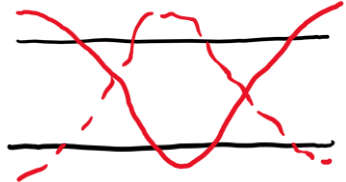
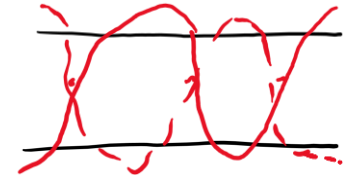
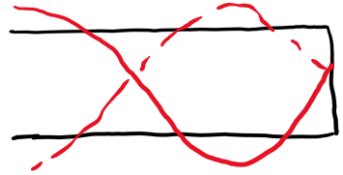
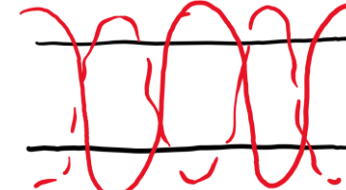
$\nwarrow$   
 $\bar{n} = 1, 2, 3, \dots$

|                         |                                                                       |                                                   |
|-------------------------|-----------------------------------------------------------------------|---------------------------------------------------|
| resonant<br>wave length | $\lambda_n = \frac{2L}{n}$                                            | $\lambda_n = \frac{4L}{2\bar{n}-1}$<br>$n$        |
| resonant<br>frequency   | $f_n = \frac{v}{\lambda} = n \frac{v}{2L} \quad (n = 1, 2, 3, \dots)$ | $f_n = n \frac{v}{4L} \quad (n = 1, 3, 5, \dots)$ |

# Resonant frequencies

closed end  $\rightarrow$  node  
open end  $\rightarrow$  antinode

odd

| $n$ | $\lambda_n = \frac{2L}{n}$ |                                                                                       | $n$ | $\lambda_n = \frac{4L}{n}$ |                                                                                       |
|-----|----------------------------|---------------------------------------------------------------------------------------|-----|----------------------------|---------------------------------------------------------------------------------------|
| 1   | $\lambda_1 = \frac{2L}{1}$ |    | 1   | $\lambda_1 = \frac{4L}{1}$ |    |
| 2   | $\lambda_2 = \frac{2L}{2}$ |    | 2   | missing                    |    |
| 3   | $\lambda_3 = \frac{2L}{3}$ |  | 3   | $\lambda_3 = \frac{4L}{3}$ |  |
| 4   | $\lambda_4 = \frac{2L}{4}$ |  | 4   | missing                    |  |

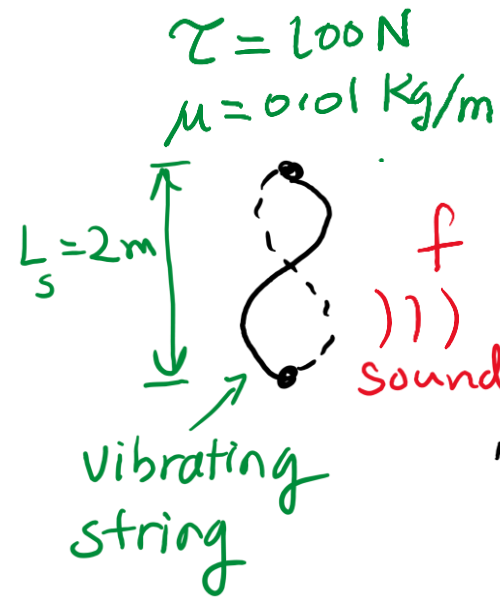
The first harmonic  
(The fundamental mode)

The second harmonic

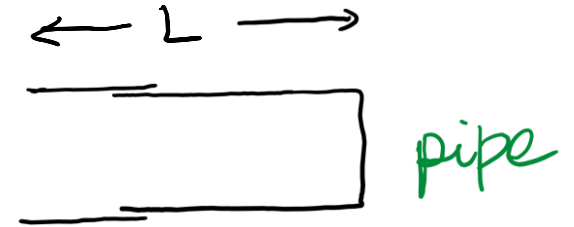
The third harmonic

The fourth harmonic

## Example



$f$   
)))  
sound



What is  $L$  to excite the fundamental mode by the string?

for the string

$$f = n \frac{v_s}{2L_s} = n \sqrt{\frac{T}{\mu}}$$

$$f = 2 \frac{\sqrt{\frac{100}{0.01}}}{2(2)} = 50 \text{ Hz}$$

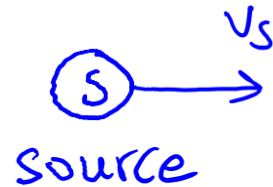
for the pipe

$$f = n \frac{v}{4L} = 1 \frac{343}{4L}$$

$$50 = \frac{343}{4L} \Rightarrow L = \frac{343}{4(50)} \text{ m}$$

## 17.7 Doppler effect

The Doppler effect is the change in the detected frequency because of a relative motion between the source of sound and the detector.



$f$  = frequency emitted  
by the source

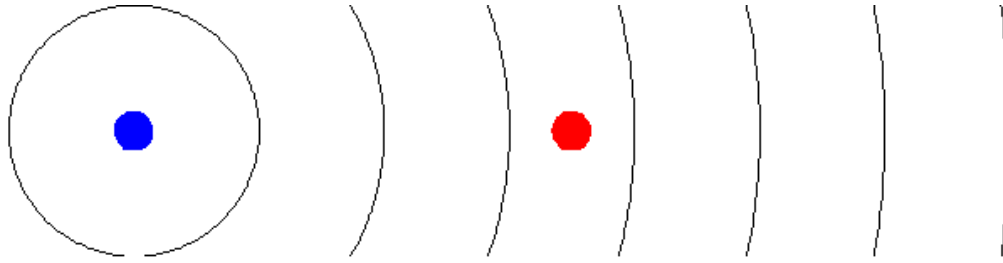
Example Loud speaker  
siren



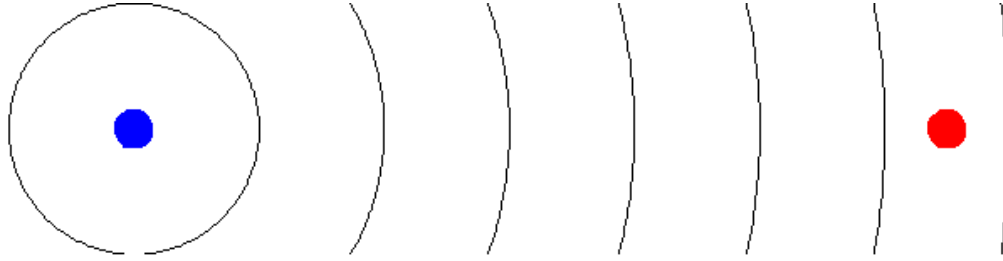
$f'$  = frequency detected  
by the detector

Example microphone  
ear

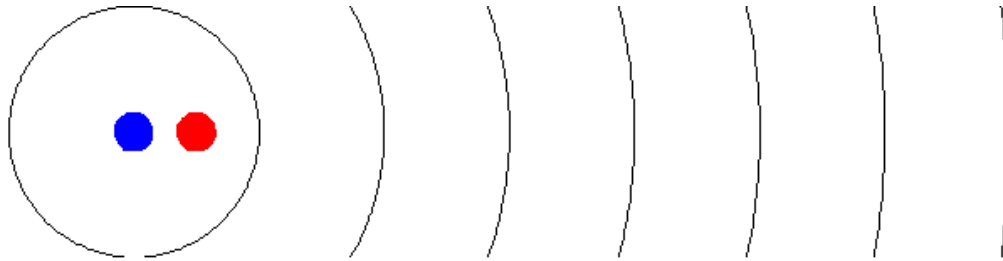
$v_s$  and  $v_d$  are measured relative to the air.



$$f' = f$$



$$f' > f$$



$$f' < f$$



$$f' = f$$



$$f' > f$$



$$f' < f$$

$$f' = f \frac{v \pm v_D}{v \pm v_S}$$

Speed of sound  $\swarrow$   
 Speed of detector relative to air  $\swarrow$   
 frequency detected by the detector  $\nearrow$   
 frequency emitted by the source  $\nearrow$   
 speed of source relative to air  $\nwarrow$

Choose the sign such that when the source or detector moves toward the other, you get a higher frequency, and when the source or detector moves away from the other, you get a lower frequency.

Toward  $\longrightarrow$  higher  $f'$

Away  $\longrightarrow$  lower  $f'$

$$f' = f \frac{v \pm v_D}{v \pm v_S}$$



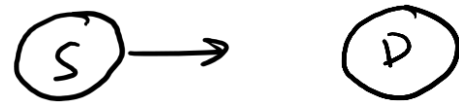
$$f' = f \frac{v - v_D}{v}$$



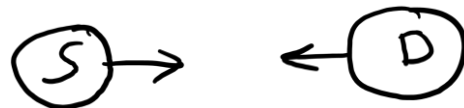
$$f' = f \frac{v + v_D}{v}$$



$$f' = f \frac{v}{v + v_S}$$



$$f' = f \frac{v}{v - v_S}$$



$$f' = f \frac{v + v_D}{v - v_S}$$

## Question

Compare  $f$  and  $f'$



$$f' > f$$



$$f' = f \frac{v - v_D}{v - v_s}$$

need to know  $v_D$  and  $v_s$  to compare.

## Example

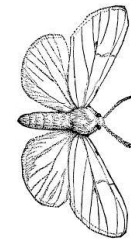
Bat



$$v_b = 13 \text{ m/s} \rightarrow$$

$$f_{be} = 66 \text{ kHz}$$

Moth



$$v_m = 8 \text{ m/s} \rightarrow$$

$$f_{md} = ?$$

What frequency  $f_{md}$  does the moth detect?

$$f' = f \frac{v \pm v_d}{v \pm v_s}$$

$$f_{md} = f_{be} \frac{v \ominus v_m}{v \ominus v_b}$$

Detector moves away

source moves toward

$$f_{md} = 66 \times 10^3 \frac{343 - 8}{343 - 13} = 67 \text{ kHz}$$

## Example

Bat

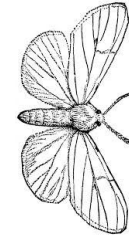


$$v_b = 13 \text{ m/s}$$

$$f_{be} = 66 \text{ kHz}$$

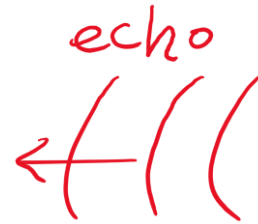
$$f_{bd} = ?$$

Moth



$$v_m = 8 \text{ m/s}$$

$$f_{md} = 67 \text{ kHz} \\ = f_{me}$$



What frequency  $f_{bd}$  does the bat detect?

$$f' = f \frac{v \pm v_d}{v \pm v_s}$$

$$f_{bd} = f_{me} \frac{v + v_b}{v + v_m}$$

detector moves toward

source moves away

$$f_{bd} = 67 \times 10^3 \frac{343 + 13}{343 + 8} = 68 \text{ kHz}$$