Q1 M1-081-01

The displacement of a string carrying a traveling sinusoidal wave is given by:

$$y(x,t) = y_m \sin(kx - \omega t + \varphi).$$

At time $t = 0$ the point at $x = 0$ has a displacement of zero and is moving in the positive y direction. Find the value of the phase constant φ.

A) 180 degrees
B) 90 degrees
C) 135 degrees
D) 0 degrees
E) 270 degrees

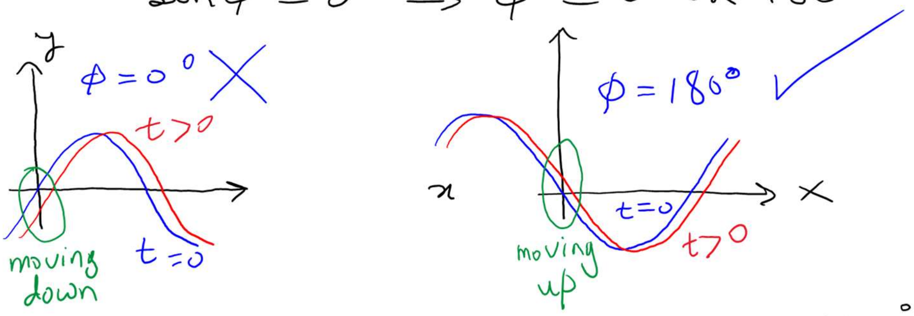
moving to the right

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$y(x=0,t=0) = y_m \sin\phi = 0$$

$$\sin\phi = 0 \Rightarrow \phi = 0° \text{ or } 180°$$

One way



Another way

$$u(x,t) = -y_m \omega \cos(kx - \omega t + \phi)$$

$$\Rightarrow u(x=0,t=0) = -y_m \omega \cos\phi = \begin{cases} -y\omega\cos 0° = -y\omega \\ -y\omega\cos 180° = y\omega \end{cases}$$

moving up $u > 0 \Rightarrow \phi = 180°$

Q2 M1-082-04

A transverse sinusoidal wave is travelling on a stretched string. The maximum transverse speed of a particle on the string is 24.0 m/s. The frequency of oscillations of a particle in the string is 120 Hz. What is the amplitude of the wave?

A) 31.8 mm
B) 25.1 mm
C) 12.0 mm
D) 43.3 mm
E) 53.2 mm

$$u_{max} = y_m \omega = y_m 2\pi f$$

$$\Rightarrow y_m = \frac{u_{max}}{2\pi f} = \frac{24}{2\pi \times 120} = 0.0381 \text{ m} = 38.1 \text{ mm}$$

Q3 M1-081-02

A stretched string of mass 2.0 g and length 10 cm, carries a wave having the following displacement wave: $y(x,t) = 0.05 \sin(2\pi x - 400\pi t)$, where x and y are in meters and t is in seconds. What is the tension in the string?

A) 800 N
B) 150 N
C) 55 N
D) 15 N
E) 100 N

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = v^2 \mu = \left(\frac{\omega}{k}\right)^2 \mu = \left(\frac{\omega}{k}\right)^2 \frac{m}{l}$$

$$\tau = \left(\frac{400\pi}{2\pi}\right)^2 \frac{2 \times 10^{-3}}{10 \times 10^{-2}} = 800 \text{ N}$$

Q4 M1-092-02

A transverse sinusoidal travelling wave on a stretched string is given by:
y (x,t) = 0.00230 *sin* (6.98 *x* + 742 *t*), where *x* and *y* are in meters, and *t* is in seconds. The length of the string is 1.35 m and its mass is 3.38 g. What is the average power carried by the wave?

A) 0.387 W
B) 0.774 W
C) 0.194 W
D) 0.457 W
E) 0.513 W

$$P_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2 = \frac{1}{2} \frac{3.38\times10^{-3}}{1.35} \frac{742}{6.98} 742^2 (0.0023)^2$$
$$= 0.388 \text{ W}$$

Q5 M1-092-03

Two identical sinusoidal waves travel simultaneously in the same direction along the same string. Each wave has an amplitude of y_m. If the amplitude of the resultant wave is $y_m/2$, what is the phase difference between the two waves?

A) 151°
B) 75.5°
C) 120°
D) 60.0°
E) 110°

$$2y_m \cos \frac{1}{2}\phi = \frac{y_m}{2}$$
$$\Rightarrow \cos\frac{1}{2}\phi = \frac{1}{4} \Rightarrow \frac{1}{2}\phi = \cos^{-1}\frac{1}{4}$$
$$\Rightarrow \phi = 2\cos^{-1}\left(\frac{1}{4}\right) = 151^\circ$$

Q6 M1-092-06

Two identical strings (same mass and length), each fixed at both ends, are arranged near each other. If string A starts oscillating in its fundamental mode, it is observed that string B will begin vibrating in its third normal mode (n = 3). What is the ratio of the tension in string B to that in string A?

A) 1/9
B) 9
C) 1/3
D) 3
E) 1

$$f_n = n\frac{v}{2L}$$

For string A $\quad f_{A1} = 1\frac{v_A}{2L}$

For string B $\quad f_{B3} = 3\frac{v_B}{2L}$

$$f_{A1} = f_{B3} \Rightarrow \frac{v_A}{2L} = 3\frac{v_B}{2L}$$

$$v_A = 3 v_B$$

$$\sqrt{\frac{\tau_A}{\mu}} = 3\sqrt{\frac{\tau_B}{\mu}}$$

$$\tau_A = 9\,\tau_B$$

$$\Rightarrow \frac{\tau_B}{\tau_A} = \frac{1}{9}.$$