

A force $\vec{F} = (3\hat{i} + \hat{j})$ N acts on a particle located at $\vec{r} = (2\hat{i} + 4\hat{j})$ m. What is the torque about the origin?

A. $(12\hat{i} + 2\hat{j})$ N*m

B. $-10\hat{k}$ N*m

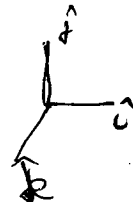
C. $+10\hat{k}$ N*m

D. $(5\hat{i} + 5\hat{j})$ N*m

E. $(6\hat{i} + 4\hat{j})$ N*m

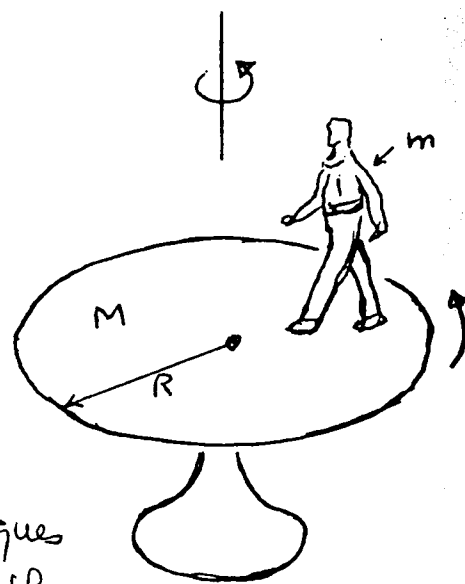
$$\vec{\tau} = \vec{r} \times \vec{F} = (2\hat{i} + 4\hat{j}) \times (3\hat{i} + \hat{j})$$

$$= 2\hat{k} - 12\hat{k} = -10\hat{k} \text{ (N.m)}$$



or: $\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 3 & 1 & 0 \end{vmatrix} = 0\hat{i} - 0\hat{j} - 10\hat{k} \text{ (N.m)}$

A horizontal platform in the shape of a circular disk rotates in a horizontal plane about a frictionless vertical axle (see figure). The platform has a mass of 100 kg and a radius of 2 m. A student whose mass is 60 kg walks slowly from the rim of the platform toward the center. If the angular velocity of the system is 2 rad/sec when the student is at the rim, calculate the angular velocity when the student has reached a point 0.5 m from the center. (take $I_{\text{platform}} = \frac{1}{2}MR^2$)



A. 10.5 rad/sec

B. 4.1 rad/sec

C. 8.2 rad/sec

D. 6.3 rad/sec

E. 12.3 rad/sec

Since no external torques act on the system, the

$$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f$$

$$\left(\frac{1}{2}MR^2 + mR^2\right)\omega_i = \left(\frac{1}{2}MR^2 + mr^2\right)\omega_f$$

$$\left[\frac{1}{2}(100)(2)^2 + 60(2)^2\right](2) = \left[\frac{1}{2}(100)(2)^2 + 60(0.5)^2\right]\omega_f$$

$$\Rightarrow \omega_f = 4.1 \text{ rad/s}$$

Which of the following statements is INCORRECT ?

- A. The angular momentum of a particle depends on the choice of the origin about which it is defined.
- (B) For a given constant torque, the angular acceleration of a rigid body is directly proportional to its moment of inertia.
- C. The moment of inertia of a rigid body depends on the axis about which it is defined.
- D. The center of gravity of a large rigid body coincides with its center of mass if "g" is constant over its mass distribution.
- E. The torque of a force acting on a rigid body about a given point P is Zero if the line of action of the force passes through point P.

A force of magnitude 10.0 N acts on a rigid body. The force lies in the X-Y plane. Its line of action passes through the point (0.30, 0.00) m and makes an angle of 30.0 degrees with the + X-axis. What is the torque of the force about the point (-0.20, 0.00) m ?

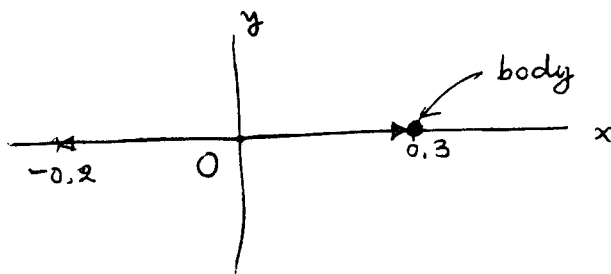
- A. +2.50 * k N.m
- B. -6.25 * j N.m
- C. +6.25 * j N.m
- D. -2.50 * k N.m
- E. +3.75 * i N.m

$$\vec{F} = 10 \cos 30^\circ \hat{i} + 10 \sin 30^\circ \hat{j} = 8.7 \hat{i} + 5 \hat{j}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = 0.5 \hat{i} + 0 \hat{j}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0 \\ 8.7 & 5 & 0 \end{vmatrix} = 0 \hat{i} - 0 \hat{j} + 2.5 \hat{k}$$

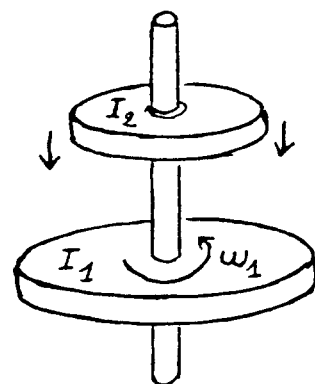
$$\vec{\tau} = 2.5 \hat{k} \text{ N.m}$$



Four statements out of the following five are correct and only one is incorrect. Which statement is NOT correct ?

- (A.) The angular momentum of a particle does not depend on the choice of the origin about which it is defined.
- B. The gravitational mass of a body on earth is equal to its gravitational mass on the moon.
- C. If a particle is moving with constant velocity, then the resultant external force on it must be zero.
- D. The moment of inertia of a rigid body depends on the axis about which it is defined.
- E. The acceleration of a particle is always in the same direction as the resultant external force.

A disk of rotational inertia I_1 is rotating freely with angular speed " ω " when a second, non-rotating disk with rotational inertia $I_2 = (1/2) * I_1$, is dropped on it (see figure). The two then rotate as a unit. What is the final angular speed ?



- (A.) $(2/3) * \text{"omega"}$
- B. $(1/2) * \text{"omega"}$
- C. $2.0 * \text{"omega"}$
- D. $(3/4) * \text{"omega"}$
- E. $(1/3) * \text{"omega"}$

$$I_i \omega_i = I_f \omega_f$$

$$I_1 \omega_1 = (I_1 + I_2) \omega_f$$

$$I_1 \omega_1 = \frac{3}{2} I_1 \omega_f$$

$$\Rightarrow \omega_f = \frac{2}{3} \omega_1$$

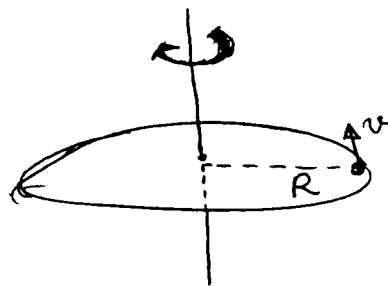
A solid sphere has a radius of 0.2 m and a mass of 150 kg. How much work is required to get the sphere rolling with an angular speed of 50 rad/sec on a horizontal surface ? (assume the sphere starts from rest and rolls without slipping; $I_{\text{sphere}} = \frac{2}{5} * M * R^2$).

- A. $7.03 * 10^{**2} \text{ J}$
- B. $5.35 * 10^{**3} \text{ J}$
- C. $6.25 * 10^{**3} \text{ J}$
- D. $3.35 * 10^{**4} \text{ J}$
- (E.) $1.05 * 10^{**4} \text{ J}$

$$\begin{aligned}
 W = \Delta K &= K_f - K_i \\
 &= \frac{1}{2} I_{\text{cm}} \omega^2 + \frac{1}{2} m v_{\text{cm}}^2 \\
 &= \frac{1}{2} \frac{2}{5} m R^2 \frac{v_{\text{cm}}^2}{R^2} + \frac{1}{2} m v_{\text{cm}}^2 \\
 &= \frac{7}{10} m v_{\text{cm}}^2 = \frac{7}{10} m \omega^2 R^2 \\
 &= 10500 \text{ J}
 \end{aligned}$$

A man of mass 80.0 kg stands at the rim of a turntable of radius 2.0 m. The moment of inertia of the turntable is $4000 \text{ kg} \cdot \text{m}^2$ and it is mounted on a vertical frictionless shaft through its center. The whole system is initially at rest. Now the man starts walking along the outer edge of the turntable with a speed of 1.0 m/s relative to the earth. Calculate the angular speed of the turntable. You may consider the man as a point mass.

- A. 0.08 radians/sec
- B. 0.12 radians/sec
- C. 0.05 radians/sec
- D. 0.06 radians/sec
- ☒ E. 0.04 radians/sec



Since no external torques,
the angular momentum of the
system is conserved $\Rightarrow L_i = L_f$

Since the system is initially at rest $L_i = 0$

$$L_f = +L_{\text{table}} + L_{\text{man}} = -I\omega + mvr = 0$$

$$\Rightarrow \omega = \frac{mvr}{I} = \frac{(80)(1)(2)}{4000} = 0.04 \frac{\text{rad}}{\text{s}}$$

A solid sphere of mass 10 kg rolls without slipping on a rough surface. At the instant its center of mass has a speed of 10 m/s, determine the total kinetic energy of the sphere.

- A. 250 J
- B. 400 J
- ☒ C. 700 J
- D. 750 J
- E. 500 J

$$K = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega^2 \quad \omega = \frac{v_{\text{cm}}}{R}$$

$$= \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} \left(\frac{2}{5} m R^2 \right) \left(\frac{v_{\text{cm}}^2}{R^2} \right)$$

$$= \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{5} m v_{\text{cm}}^2 = \frac{7}{10} m v_{\text{cm}}^2$$

$$K = 700 \text{ J}$$

A man stands on a frictionless rotating platform that is rotating with a speed of 1.0 rev/sec. His arms are outstretched and he holds a weight in each hand. With his hands in this position the moment of inertia of the man, the weight and the platform is $6.0 \text{ kg}\cdot\text{m}^2$. If, by drawing in the weights the man decreases the moment of inertia to $2.0 \text{ kg}\cdot\text{m}^2$, what is the resulting angular speed of the platform? ($\pi = 3.1416$)

- A. $8\pi \text{ rad/s}$
- ☒ B. $6\pi \text{ rad/s}$
- C. 2.0 rad/s
- D. $3\pi \text{ rad/s}$
- E. 3.0 rad/s

$L_i = L_f$ because no external torque act on the system

$$\Rightarrow I_i \omega_i = I_f \omega_f$$

$$\Rightarrow \omega_f = \frac{I_i \omega_i}{I_f} = \frac{6 \times 1 \times 2 \times \pi}{2} = 6\pi \text{ rad/s}$$

An acrobat plane was spinning at 2 rad/s when both its wings broke off. If its new moment of inertia is 4 times smaller, what is its new angular velocity?

- A. 0
- B. 2 rad/s
- ☒ C. 8 rad/s
- D. 16 rad/s
- E. 0.5 rad/s

because no external torques act on the system then

$$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f$$

$$\omega_f = \frac{I_i \omega_i}{I_f} = 4 \omega_i = 8 \text{ rad/s}$$

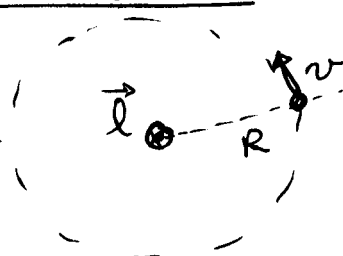
A 0.010-kg particle moves in a circular path of radius 10 m in the plane of the paper. Find the magnitude and direction of its angular momentum about the axis through the centre, and perpendicular to the plane of the circle when its velocity is 20 m/s in the counter-clockwise direction.

- A. 2.0 kg*m**2/s , out of the paper
- B. 2.0 kg*m**2/s , into the paper
- C. 40 kg*m**2/s , into the paper
- D. 20 kg*m**2/s , normal to the velocity
- E. 40 kg*m**2/s , out of the paper

$$l = m v r_{\perp} = (0.01)(20)(10)$$

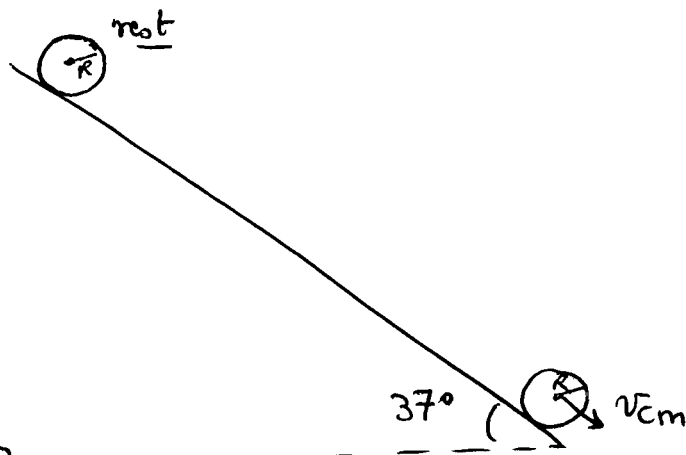
$$= 2 \text{ kg} \cdot \frac{\text{m}^2}{\text{s}} \text{ out of the paper.}$$

overhead view



A solid cylinder of mass M and radius R rolls from rest down an inclined plane of inclination angle 37 deg. Find the speed of its center of mass when the cylinder has rolled 5.00 m down the inclined plane. ($I = \frac{1}{2} * M * R^2$)

- A. 8.37 m/s
- B. 5.39 m/s
- C. 7.35 m/s
- D. 6.27 m/s
- E. 9.34 m/s



$$\Delta K + \Delta U_g = 0$$

$$\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 - mgL \sin \theta = 0$$

$$\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \frac{v_{cm}^2}{R^2} = mgL \sin \theta$$

$$v_{cm} = \sqrt{\frac{2mgL \sin \theta}{m + \frac{I}{R^2}}} = \sqrt{\frac{4gL \sin \theta}{3}}$$

$$= 6.27 \text{ m/s}$$