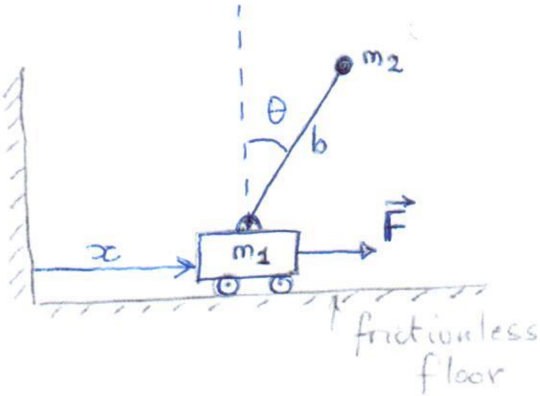
PHYS 301 – Term 151
Quiz#5-chapter 7
19/11/2015

Name: Key　　　　　　　　　　　　　　　Id#:

Consider the figure shown. The block has mass m_1, and a constant force F is applied to the right. The rod is massless and has length b. The bob has mass m_2. The system is moving to the right. Assume the force F is conservative.

(a) Use the generalized coordinates shown in the figure to write the **kinetic energy of the system**.
(b) Write the **potential energy of the system**.
(c) Write the **Lagrangian of the system**.
(d) Write the **Lagrange equations**.



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x} + b\dot{\theta}\cos\theta)^2 + \frac{1}{2} m_2 (b\dot{\theta}\sin\theta)^2$$

$$U = mgb\cos\theta - Fx$$

$$L = T - U = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x} + b\dot{\theta}\cos\theta)^2 + \frac{1}{2} m_2 (b\dot{\theta}\sin\theta)^2 - mgb\cos\theta + Fx$$

$$\frac{\partial L}{\partial x} = F \qquad \frac{\partial L}{\partial \dot{x}} = (m_1 + m_2)\dot{x} + m_2 b\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m_1 + m_2)\ddot{x} + m_2 b\ddot{\theta}\cos\theta - m_2 b\dot{\theta}^2\sin\theta$$

$$\boxed{(m_1 + m_2)\ddot{x} + m_2 b\cos\theta\,\ddot{\theta} - m_2 b\sin\theta\,\dot{\theta}^2 - F = 0}$$

$$\frac{\partial L}{\partial \theta} = -m_2 \dot{x} b\dot{\theta}\sin\theta + m_2 g b\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_2 b^2\dot{\theta} + m_2 \dot{x} b\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m_2 b^2\ddot{\theta} + m_2 \ddot{x} b\cos\theta - m_2 \dot{x} b\dot{\theta}\sin\theta$$

$$m_2 b^2\ddot{\theta} + m_2\ddot{x} b\cos\theta - \cancel{m_2 \dot{x} b\dot{\theta}}\sin\theta + \cancel{m_2 \dot{x} b\dot{\theta}}\sin\theta - m_2 g b\sin\theta = 0$$

$$\boxed{\ddot{\theta} + \ddot{x}\frac{\cos\theta}{b} - \frac{g}{b}\sin\theta = 0}$$