PHYS 301 – Term 151

Quiz#1-chapter 2

08/09/2015

Name: Key Id#:

A particle of mass m is acted upon by a one dimensional force whose potential energy is given by

$$U(x) = ax^2 - bx^3$$

where a and b are positive constants.

(a) Find the force producing this potential.

(b) Find the equilibrium positions, and state whether they are stable or unstable.

(c) Sketch the potential energy curve.

(d) Discuss the motion for $E_1 = 2E_0$, $E_2 = E_0/2$, and $E_3 = -E_0$ where $E_0 = 4a^3/27b^2$.

(a) $\vec{F} = -\vec{\nabla} U$

one dimensional problem $F = -\dfrac{\partial U}{\partial x} = -2ax + 3bx^2$

$$\boxed{F(x) = -2ax + 3bx^2}$$

(b) Equilibrium $\Rightarrow \dfrac{\partial U}{\partial x} = 0 \Rightarrow 2ax - 3bx^2 = 0$

$x(2a - 3bx) = 0 \Rightarrow$ $x = 0$ or $x = \dfrac{2a}{3b}$

$\underline{x = 0}$ $\dfrac{\partial^2 U(0)}{\partial x^2} = 2a - 6bx\Big|_{x=0} = 2a > 0$ stable

$x = \dfrac{2a}{3b}$ $\dfrac{\partial^2 U(\frac{2a}{3b})}{\partial x^2} = 2a - 6bx\Big|_{x=\frac{2a}{3b}} = 2a - 6b\left(\dfrac{2a}{3b}\right)$

$= 2a - 4a$

$= -2a < 0$

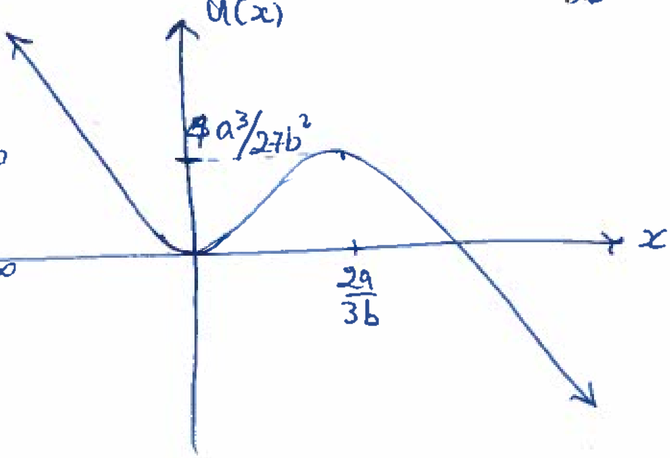
unstable

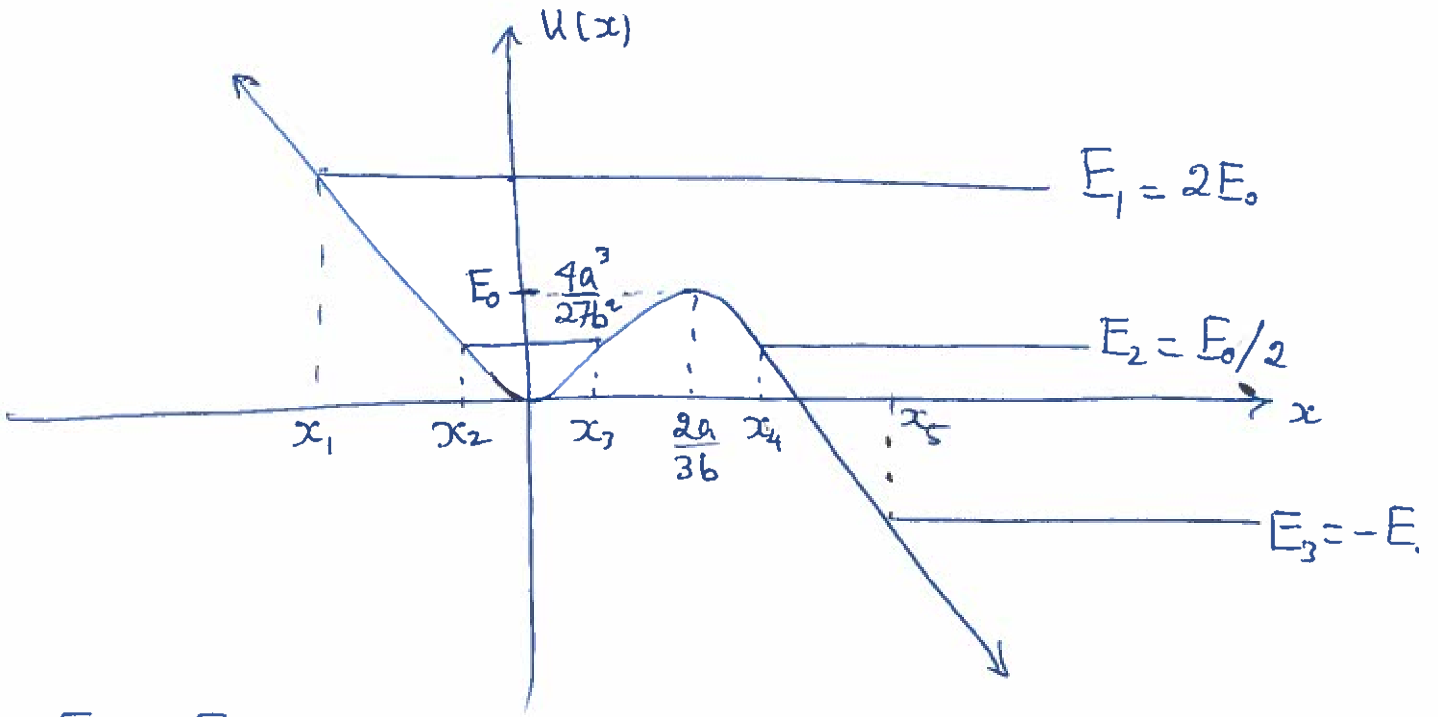
(c)

$x \to +\infty$ $U(x) \to -\infty$

$x \to -\infty$ $U(x) \to +\infty$

$x \to 0$ $U(x) \to 0$





- $E = E_1 = 2E_0$

  The partice comes from $+\infty$, reaches position x_1 where the velocity $= 0$, and returns back like a ball hitting a wall. The region $x < x_1$ is forbidden.

- $E = E_2 = E_0/2$

  The particle oscillates between x_2 and x_3

  The particle comes from $+\infty$, reaches position x_4 where the velocity is zero, and returns back like a ball hitting the wall. $x < x_2$ is forbidden.

- $E = E_3 = -E_0$

  The particle comes from ∞, reaches position x_5 where the velocity is zero and returns back like a ball hitting a wall. The region $x < x_5$ is forbidden.