6. When we say that a moving clock runs slower than a stationary one, does this imply that there is something physically unusual about the moving clock?
7. When we speak of time dilation, do we mean that time passes more slowly in moving systems or that it simply appears to do so?
8. List some ways our day-to-day lives would change if the speed of light were only 50 m/s.
9. Give a physical argument to show that it is impossible to accelerate an object of mass m to the speed of light, even with a continuous force acting on it.
10. It is said that Einstein, in his teenage years, asked the question, "What would I see in a mirror if I carried it in my hands and ran at the speed of light?" How would you answer this question?
11. Suppose astronauts were paid according to the time spent traveling in space. After a long voyage at a speed near that of light, a crew of astronauts returns and opens their pay envelopes. What will their reaction be?
12. What happens to the density of an object as its speed increases, as measured by an Earth observer?

PROBLEMS

1.2 The Principle of Newtonian Relativity and the Galilean Transformation

1. In a lab frame of reference, an observer finds Newton's second law is valid in the form $\sum_{\substack{\text{actual}\\\text{physical}\\\text{forces}}} F = ma$. Show that Newton's second law is not valid in a reference frame moving past the laboratory frame of Problem 1 with a constant acceleration a_1. Assume that mass is an invariant quantity and is constant in time.
2. A 2000-kg car moving with a speed of 20 m/s collides with and sticks to a 1500-kg car at rest at a stop sign. Show that because momentum is conserved in the rest frame, momentum is also conserved in a reference frame moving with a speed of 10 m/s in the direction of the moving car.
3. A billiard ball of mass 0.3 kg moves with a speed of 5 m/s and collides elastically with a ball of mass 0.2 kg moving in the opposite direction with a speed of 3 m/s. Show that because momentum is conserved in the rest frame, it is also conserved in a frame of reference moving with a speed of 2 m/s in the direction of the second ball.

1.3 The Michelson–Morley Experiment

4. An airplane flying upwind, downwind, and crosswind shows the main principle of the Michelson–Morley experiment. A plane capable of flying at speed c in still air is flying in a wind of speed v. Suppose the plane flies upwind a distance L and then returns downwind to its starting point. (a) Find the time needed to make the round-trip and compare it with the time to fly crosswind a distance L and return. Before calculating these times, sketch the two situations. (b) Compute the time difference for the two trips if $L = 100$ mi, $c = 500$ mi/h, and $v = 100$ mi/h.

1.5 Consequences of Special Relativity

5. With what speed will a clock have to be moving in order to run at a rate that is one-half the rate of a clock at rest?
6. How fast must a meter stick be moving if its length is observed to shrink to 0.5 m?
7. A clock on a moving spacecraft runs 1 s slower per day relative to an identical clock on Earth. What is the relative speed of the spacecraft? (*Hint:* For $v/c \ll 1$, note that $\gamma \approx 1 + v^2/2c^2$.)
8. A meter stick moving in a direction parallel to its length appears to be only 75 cm long to an observer. What is the speed of the meter stick relative to the observer?
9. A spacecraft moves at a speed of $0.900c$. If its length is L as measured by an observer on the spacecraft, what is the length measured by a ground observer?
10. The average lifetime of a pi meson in its own frame of reference is 2.6×10^{-8} s. If the meson moves with a speed of $0.95c$, what is (a) its mean lifetime as measured by an observer on Earth and (b) the average distance it travels before decaying, as measured by an observer on Earth?
11. An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600 s when the jet moves with a speed of 400 m/s. How much longer or shorter a time interval does an identical clock held by an observer on the ground measure? (*Hint:* For $v/c \ll 1$, $\gamma \approx 1 + v^2/2c^2$.)
12. An astronaut at rest on Earth has a heartbeat rate of 70 beats/min. What will this rate be when she is traveling in a spaceship at $0.90c$ as measured (a) by an observer also in the ship and (b) by an observer at rest on the Earth?
13. The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_0, the number at time t is given by $N = N_0 e^- t/\tau$, where τ is the mean lifetime, equal to 2.2 μs. Suppose the muons move at a speed of $0.95c$ and there are 5.0×10^4 muons at $t = 0$. (a) What is the observed lifetime of the muons? (b) How many muons remain after traveling a distance of 3.0 km?
14. A rod of length L_0 moves with a speed v along the horizontal direction. The rod makes an angle of θ_0 with

respect to the x'-axis. (a) Show that the length of the rod as measured by a stationary observer is given by $L = L_0[1 - (v^2/c^2)\cos^2 \theta_0]^{1/2}$. (b) Show that the angle that the rod makes with the x-axis is given by the expression $\tan \theta = \gamma \tan \theta_0$. These results show that the rod is both contracted and rotated. (Take the lower end of the rod to be at the origin of the primed coordinate system.)

15. *The classical Doppler shift for light.* A light source recedes from an observer with a speed v that is small compared with c. (a) Show that in this case, Equation 1.15 reduces to

$$\frac{\Delta f}{f} \approx -\frac{v}{c}$$

(b) Also show that in this case

$$\frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$$

(*Hint:* Differentiate $\lambda f = c$ to show that $\Delta\lambda/\lambda = -\Delta f/f$) (c) Spectroscopic measurements of an absorption line normally found at $\lambda = 397$ nm reveal a redshift of 20 nm for light coming from a galaxy in Ursa Major. What is the recessional speed of this galaxy?

16. Calculate, for the judge, how fast you were going in miles per hour when you ran the red light because it appeared Doppler-shifted green to you. Take red light to have a wavelength of 650 nm and green to have a wavelength of 550 nm.

17. (a) How fast and in what direction must galaxy A be moving if an absorption line found at 550 nm (green) for a stationary galaxy is shifted to 450 nm (blue) for A? (b) How fast and in what direction is galaxy B moving if it shows the same line shifted to 700 nm (red)?

18. Police radar detects the speed of a car (Fig. P1.18) as follows: Microwaves of a precisely known frequency are broadcast toward the car. The moving car reflects the microwaves with a Doppler shift. The reflected waves are received and combined with an attenuated version of the transmitted wave. Beats occur between the two microwave signals. The beat frequency is measured. (a) For an electromagnetic wave reflected back to its source from a mirror approaching at speed v, show that the reflected wave has frequency

$$f = f_{\text{source}} \frac{c + v}{c - v}$$

where f_{source} is the source frequency. (b) When v is much less than c, the beat frequency is much smaller than the transmitted frequency. In this case use the approximation $f + f_{\text{source}} \approx 2 f_{\text{source}}$ and show that the beat frequency can be written as $f_{\text{beat}} = 2v/\lambda$. (c) What beat frequency is measured for a car speed of 30.0 m/s if the microwaves have frequency 10.0 GHz? (d) If the beat frequency measurement is accurate to ± 5 Hz, how accurate is the velocity measurement?

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1.6 The Lorentz Transformation

19. Two spaceships approach each other, each moving with the *same* speed as measured by an observer on the Earth. If their *relative* speed is $0.70c$, what is the speed of each spaceship?

20. An electron moves to the right with a speed of $0.90c$ relative to the laboratory frame. A proton moves to the right with a speed of $0.70c$ relative to the electron. Find the speed of the proton relative to the laboratory frame.

21. An observer on Earth observes two spacecraft moving in the *same* direction toward the Earth. Spacecraft A appears to have a speed of $0.50c$, and spacecraft B appears to have a speed of $0.80c$. What is the speed of spacecraft A measured by an observer in spacecraft B?

22. *Speed of light in a moving medium.* The motion of a medium such as water influences the speed of light. This effect was first observed by Fizeau in 1851. Consider a light beam passing through a horizontal column of water moving with a speed v. (a) Show that if the beam travels in the same direction as the flow of water, the speed of light measured in the laboratory frame is given by

$$u = \frac{c}{n}\left(\frac{1 + nv/c}{1 + v/nc}\right)$$

where n is the index of refraction of the water. (*Hint:* Use the inverse Lorentz velocity transformation and note that the speed of light with respect to the moving frame is given by c/n.) (b) Show that for $v \ll c$, the preceding expression is in good agreement with Fizeau's experimental result:

$$u \approx \frac{c}{n} + v - \frac{v}{n^2}$$

This proves that the Lorentz velocity transformation and not the Galilean velocity transformation is correct for light.

23. An observer in frame S sees lightning simultaneously strike two points 100 m apart. The first strike occurs at $x_1 = y_1 = z_1 = t_1 = 0$ and the second at $x_2 = 100$ m, $y_2 = z_2 = t_2 = 0$. (a) What are the coordinates of these two events in a frame S′ moving in the standard configuration at $0.70c$ relative to S? (b) How far apart are the events in S′? (c) Are the events simultaneous in S′? If not, what is the difference in time between the events, and which event occurs first?

24. As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by an Earth observer to have velocity $u_y = -0.90c$ and B to have a velocity $u_x = +0.90c$, find the speed of ship A as measured by the pilot of B.

ADDITIONAL PROBLEMS

25. In 1962, when Scott Carpenter orbited Earth 22 times, the press stated that for each orbit he aged 2 millionths of a second less than if he had remained on Earth. (a) Assuming that he was 160 km above Earth in an eastbound circular orbit, determine the time difference between someone on Earth and the orbiting astronaut for the 22 orbits. (b) Did the press report accurate information? Explain.

26. The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.35c$, determine the speed of the faster spaceship.

27. The pion has an average lifetime of 26.0 ns when at rest. For it to travel 10.0 m, how fast must it move?

28. If astronauts could travel at $v = 0.95c$, we on Earth would say it takes $(4.2/0.95) = 4.4$ years to reach Alpha Centauri, 4.2 lightyears away. The astronauts disagree. (a) How much time passes on the astronauts' clocks? (b) What distance to Alpha Centauri do the astronauts measure?

29. A spaceship moves away from Earth at a speed v and fires a shuttle craft in the forward direction at a speed v relative to the ship. The pilot of the shuttle craft launches a probe at speed v relative to the shuttle craft. Determine (a) the speed of the shuttle craft relative to Earth, and (b) the speed of the probe relative to Earth.

30. An observer in a rocket moves toward a mirror at speed v relative to the reference frame labeled by S in Figure P1.30. The mirror is stationary with respect to S. A light pulse emitted by the rocket travels toward the mirror and is reflected back to the rocket. The front of the rocket is a distance d from the mirror (as measured by observers in S) at the moment the light pulse leaves the rocket. What is the total travel time of the pulse as measured by observers in (a) the S frame and (b) the front of the rocket?

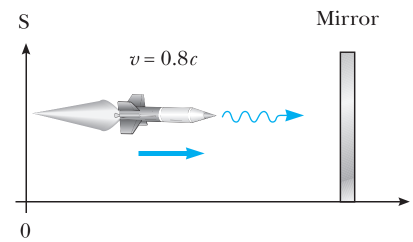

Figure P1.30

31. A physics professor on Earth gives an exam to her students who are on a spaceship traveling at speed v relative to Earth. The moment the ship passes the professor, she signals the start of the exam. If she wishes her students to have time T_0 (spaceship time) to complete the exam, show that she should wait a time (Earth time) of

$$T = T_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

before sending a light signal telling them to stop. (*Hint:* Remember that it takes some time for the second light signal to travel from the professor to the students.)

32. A yet-to-be-built spacecraft starts from Earth moving at constant speed to the yet-to-be-discovered planet Retah, which is 20 lighthours away from Earth. It takes 25 h (according to an Earth observer) for a spacecraft to reach this planet. Assuming that the clocks are synchronized at the beginning of the journey, compare the time elapsed in the spacecraft's frame for this one-way journey with the time elapsed as measured by an Earth-based clock.

33. Suppose our Sun is about to explode. In an effort to escape, we depart in a spaceship at $v = 0.80c$ and head toward the star Tau Ceti, 12 lightyears away. When we reach the midpoint of our journey from the Earth, we see our Sun explode and, unfortunately, at the same instant we see Tau Ceti explode as well. (a) In the spaceship's frame of reference, should we conclude that the

two explosions occurred simultaneously? If not, which occurred first? (b) In a frame of reference in which the Sun and Tau Ceti are at rest, did they explode simultaneously? If not, which exploded first?

34. Two powerless rockets are on a collision course. The rockets are moving with speeds of $0.800c$ and $0.600c$ and are initially 2.52×10^{12} m apart as measured by Liz, an Earth observer, as shown in Figure P1.34. Both rockets are 50.0 m in length as measured by Liz. (a) What are their respective proper lengths? (b) What is the length of each rocket as measured by an observer in the other rocket? (c) According to Liz, how long before the rockets collide? (d) According to rocket 1, how long before they collide? (e) According to rocket 2, how long before they collide? (f) If both rocket crews are capable of total evacuation within 90 min (their own time), will there be any casualties?

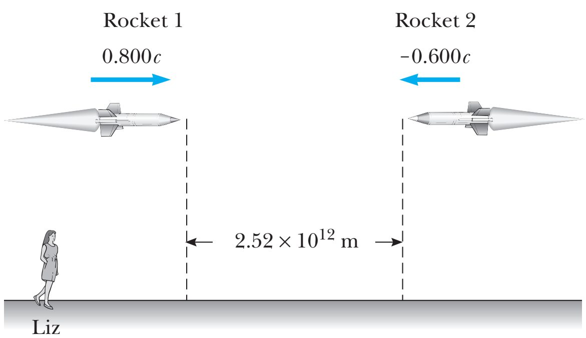

Figure P1.34

35. The identical twins Speedo and Goslo join a migration from Earth to Planet X. It is 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spaceships. Speedo's ship travels steadily at $0.950c$, and Goslo's at $0.750c$. Calculate the age difference between the twins after Goslo's spaceship reaches Planet X. Which twin is the older?

36. Suzanne observes two light pulses to be emitted from the same location, but separated in time by 3.00 μs. Mark sees the emission of the same two pulses separated in time by 9.00 μs. (a) How fast is Mark moving relative to Suzanne? (b) According to Mark, what is the separation in space of the two pulses?

37. An observer in reference frame S sees two events as simultaneous. Event A occurs at the point (50.0 m, 0, 0) at the instant 9:00:00 Universal time, 15 January 2001. Event B occurs at the point (150 m, 0, 0) at the same moment. A second observer, moving past with a velocity of $0.800c\,\hat{\mathbf{i}}$, also observes the two events. In her reference frame S′, which event occurred first and what time elapsed between the events?

38. A spacecraft is launched from the surface of the Earth with a velocity of $0.600c$ at an angle of 50.0° above the horizontal, positive x-axis. Another spacecraft is moving past with a velocity of $0.700c$ in the negative x direction. Determine the magnitude and direction of the velocity of the first spacecraft as measured by the pilot of the second spacecraft.

39. An Earth satellite used in the Global Positioning System moves in a circular orbit with period 11 h 58 min. (a) Determine the radius of its orbit. (b) Determine its speed. (c) The satellite contains an oscillator producing the principal nonmilitary GPS signal. Its frequency is 1 575.42 MHz in the reference frame of the satellite. When it is received on the Earth's surface, what is the fractional change in this frequency due to time dilation, as described by special relativity? (d) The gravitational blueshift of the frequency according to general relativity is a separate effect. The magnitude of that fractional change is given by

$$\frac{\Delta f}{f} = \frac{\Delta U_g}{mc^2}$$

where $\Delta U_g/m$ is the change in gravitational potential energy per unit mass between the two points at which the signal is observed. Calculate this fractional change in frequency. (e) What is the overall fractional change in frequency? Superposed on both of these relativistic effects is a Doppler shift that is generally much larger. It can be a redshift or a blueshift, depending on the motion of a particular satellite relative to a GPS receiver (Fig. P1.39).

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40. Show that the S′ axes, x' and ct', are nonorthogonal in a spacetime diagram. Assume that the S and S′ inertial frames move as shown in Figure 1.2 and that $t = t' = 0$ when $x = x' = 0$. (*Hint:* First use the fact that the ct'-axis is the world line of the origin of S′ to show that the ct'-axis is inclined with respect to the ct-axis. Next note that the world line of a light pulse moving in the $+x$ direction starting out at $x = 0$ and $ct = 0$ is described by the equation $x = +ct$ in S and $x' = ct'$ in S′).