

Name:

Key

ID#:

1. At what speed does a clock move if it runs at a rate which is one-half the rate of a clock at rest?

$$\Delta t = 2 \Delta t' \quad \Delta t: \text{time dilation}$$

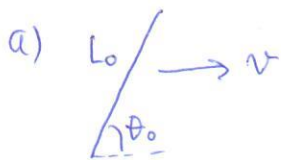
$$= \gamma \Delta t' \quad \Delta t' = \text{proper time}$$

$$\Rightarrow \gamma = 2 = \frac{1}{\sqrt{1-v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4}$$

$$v = \sqrt{\frac{3}{4}} c = \boxed{0.866c}$$

2. A rod of length $L_0 = 2.0$ m moves with speed $v = 0.98c$ along the horizontal direction. The rod makes an angle $\theta_0 = 30^\circ$ with respect to the x_0 axis.

- (a) Determine the length of the rod as measured by a stationary observer.
 (b) Determine the angle θ the rod makes with the x axis.



$$x_0 = L_0 \cos \theta_0$$

$$y_0 = L_0 \sin \theta_0$$

$$x = \frac{x_0}{\gamma} = L_0 \cos \theta_0 \sqrt{1 - v^2/c^2}$$

$$y = y_0 = L_0 \sin \theta_0$$

$$L = \sqrt{x^2 + y^2} = \sqrt{L_0^2 \cos^2 \theta_0 (1 - v^2/c^2) + L_0^2 \sin^2 \theta_0}$$

$$= \sqrt{L_0^2 - L_0^2 \cos^2 \theta_0 \frac{v^2}{c^2}} = L_0 \sqrt{1 - \cos^2 \theta_0 \frac{v^2}{c^2}}$$

$$L = 2 \sqrt{1 - \cos^2(30^\circ) (0.98)^2} = \boxed{1.06 \text{ m}}$$

b)
$$\tan \theta = \frac{y}{x} = \frac{L_0 \sin \theta_0}{L_0 \cos \theta_0 \sqrt{1 - v^2/c^2}} = \frac{\tan \theta_0}{\sqrt{1 - v^2/c^2}} = \frac{\tan 30^\circ}{\sqrt{1 - 0.98^2}}$$

$$\theta = \tan^{-1}(2.9) = \boxed{71^\circ} = 2.9$$