

37.2

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow 1-\beta^2 = \frac{1}{\gamma^2}$$

$$\beta = v/c$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} \Rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

a)  $\gamma = 1.01$

$$\beta = 0.14037076$$

b)  $\gamma = 10.0$

$$\beta = 0.99498744$$

c)  $\gamma = 100.0$

$$\beta = 0.99995000$$

d)  $\gamma = 1000.0$

$$\beta = 0.99999950$$

37.4

$$\Delta t = ? \quad v = 0.98c \quad \text{or} \quad \beta = 0.98$$

$\Delta t$  is the time dilation measured by observer in S.

$\Delta t_0$  is the proper time measured by observer in S'.

when  $\beta = 0$   $\Delta t_0 = 8$  s (from the graph). This is the proper time the clock is at rest with respect to S'.

Use the time dilation formula  $\Delta t = \gamma \Delta t_0$

when  $\beta = 0.98$       $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.98)^2}} = 5.03$

$\Rightarrow \Delta t = 5.03 \times 8 = \underline{\underline{40.2 \text{ s}}}$

37.9      $L_0 = 1.7 \text{ m}$       $v = 0.63c$

$L = \frac{L_0}{\gamma}$  (length contraction)

$L = \frac{1.7 \sqrt{1-(0.63)^2}}{1} = \underline{\underline{1.32 \text{ m}}}$

37.14      $L = \frac{L_0}{2} = \frac{L_0}{\gamma} \Rightarrow \gamma = 2$

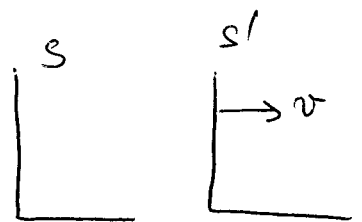
a)      $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \underline{\underline{0.866}}$

b)      $\Delta t = \gamma \Delta t_0 \Rightarrow \frac{\Delta t}{\Delta t_0} = \gamma = \underline{\underline{2}}$

37.16

$v = 0.6c$

at  $t = t' = 0$       $x' = x = 0$



in S     event 1      $(0, 0)$

event 2      $(3 \text{ km}, 4 \mu\text{s})$

a)     Lorentz transformations

in S'      $\begin{cases} x'_1 = \gamma(x - vt) = 0 \\ t'_1 = \gamma(t - \frac{vx}{c^2}) = 0 \end{cases}$      because  $x = 0$  and  $t = 0$

event 1  $\Rightarrow$

b) event 2 in  $S'$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.25$$

$$x_2' = 1.25 (3000 - 0.6 \times 3 \times 10^8 \times 4 \times 10^{-6}) = 2850 \text{ m} \\ = \underline{\underline{2.85 \text{ km}}}$$

$$t_2' = 1.25 \left( 4 \times 10^{-6} - \frac{0.6 \times 3 \times 10^8 \times 3000}{(3 \times 10^8)^2} \right)$$

$$= \underline{\underline{2.5 \times 10^{-6} \text{ s}}}$$

in  $S'$  event 2 happens before event 1 !!!  
(very strange indeed)

c) Observer in frame  $S$  sees event 1, then event 2  
Observer in frame  $S'$  sees event 2, then event 1!

37.21

a)  $v = 0.6c$       $\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = \underline{\underline{1.25}}$

b) We have two lines, one in the laboratory frame  $S$  (fixed) and one in the moving frame with the clock  $S'$ .

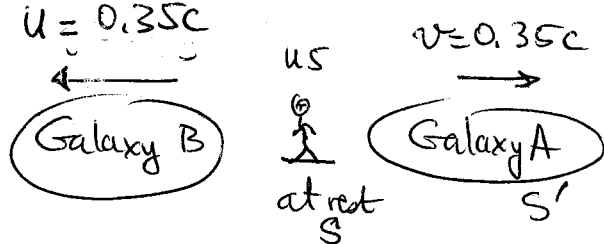
In frame  $S'$  we measure the proper time, in  $S$  we measure time dilation.

in  $S$       $d = v \Delta t \Rightarrow \Delta t = \frac{d}{v} = \frac{180}{0.6 \times 3 \times 10^8} = \underline{\underline{1 \times 10^{-6} \text{ s}}}$

in  $S'$       $\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} = \underline{\underline{0.8 \times 10^{-6} \text{ s}}}$

37.27

receding = moving away



a) Galaxy A will find the speed of our Galaxy to be receding by  $0.35c$ !

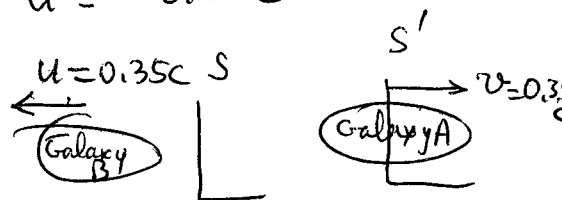
b) use Lorentz transformation for velocities

$$u' = \frac{u - v}{1 + \frac{uv}{c^2}}$$

$$v = +0.35c$$

$$u = -0.35c$$

$$u' = \frac{-0.35c - 0.35c}{1 - (-0.35)(+0.35)}$$

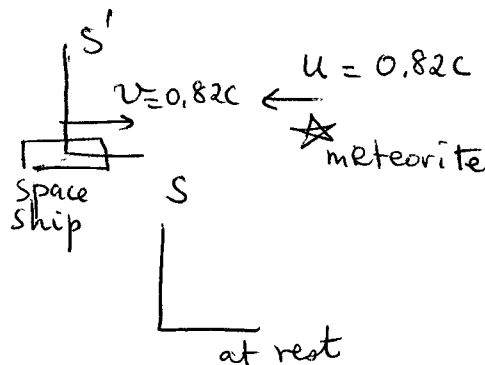


$$\frac{u'}{c} = \frac{-0.7}{1 + 0.1225} = -0.62$$

37.33

a)  $L_0 = 350\text{m}$

$v = 0.82c$



let us calculate the velocity of the meteorite as measured by the ship

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{(-0.82 - 0.82)c}{1 - (-0.82)(0.82)}$$

$$= \underline{\underline{-0.98c}} \text{ (moving to the left) see figure.}$$

proper length

this is the velocity of the meteorite wrt the ship (S' frame)

b)  $\Delta t = \frac{d}{u'} = \frac{350}{0.98c}$

$$\Delta t = 1.2 \times 10^{-6} \text{ s} = \underline{\underline{1.2 \mu\text{s}}}$$

37.35

$$v = \frac{\Delta\lambda}{\lambda} c = \frac{620\text{ nm} - 540\text{ nm}}{620\text{ nm}} \times \underbrace{3 \times 10^8}_{c} \text{ m/s}$$

$$= 0.13c = 3.87 \times 10^7 = \underline{\underline{0.387 \times 10^8 \text{ m/s}}}$$

of course this speed is unrealistic!

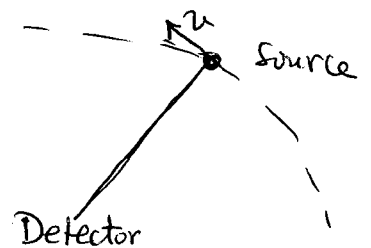
37.38

$v = 0.1c$  Doppler shift for light (transverse)

$$\lambda_0 = 589 \text{ nm}$$

$$f = f_0 \sqrt{1 - \beta^2}$$

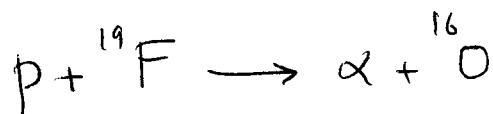
$$\frac{c}{\lambda} = \frac{c}{\lambda_0} \sqrt{1 - \beta^2}$$



$$\lambda = \frac{\lambda_0}{\sqrt{1 - \beta^2}} \Rightarrow \lambda - \lambda_0 = \lambda_0 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right)$$

$$\Delta\lambda = \lambda - \lambda_0 = 589 \left( \frac{1}{\sqrt{1 - (0.1)^2}} - 1 \right) = \underline{\underline{2.97 \text{ nm}}}$$

37.42



$$m(p) = 1.007825 \text{ u}$$

$$m(\text{F}) = 18.998405 \text{ u}$$

$$m(\alpha) = 4.002603 \text{ u}$$

$$m(\text{O}) = 15.994915 \text{ u}$$

$$Q = \Delta M c^2 = (M_i - M_f) c^2$$

$$M_i = (1.007825 + 18.998405)u = 20.00623u$$

$$M_f = (4.002603 + 15.994915)u = 19.997518u$$

$$\Delta M = 8.7 \times 10^{-3} u$$

$$c^2 = 931.494013 \frac{\text{MeV}}{u}$$

$$\Rightarrow \underline{\underline{Q = 8.12 \text{ MeV}}}$$

In this problem the final mass is less than the initial mass, the mass is changed to energy!

37.46

$$K = (\gamma - 1) m_0 c^2 \quad m_0 \text{ is the rest mass of the electron}$$

$$\frac{K}{m_0 c^2} = \gamma - 1 \Rightarrow \gamma = 1 + \frac{K}{m_0 c^2} \quad m_0 c^2 = 0.511 \text{ MeV}$$

$$a) \quad K = 1 \text{ keV} \Rightarrow \gamma = 1 + \frac{1}{511} = 1.001956947$$

$$\gamma = \underline{\underline{1.0019570}}$$

$$b) \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}} = \underline{\underline{0.062469474}}$$

$$c) \quad K = 1 \text{ MeV} \quad \gamma = 2.9569514 \quad d) \quad \beta = 0.94107924$$

$$e) \quad K = 1 \text{ GeV} \quad \gamma = 1957.9514 \quad f) \quad \beta = 0.99999987$$