$pV = nRT$ take $n = 1$ mole

$pV = RT$ differentiate $p\,dV + V\,dp = R\,dT$

$R = C_p - C_v \Rightarrow p\,dV + V\,dp = (C_p - C_v)\,dT$

$p\,dV + C_v\,dT = -V\,dp + C_p\,dT$

$dE_{int} = dQ - dW \Rightarrow nC_v\,dT = dQ - p\,dV$

$dQ = C_v\,dT + p\,dV$ for adiabatic $dQ = 0$

$\Rightarrow \quad p\,dV + C_v\,dT = 0 \quad \& \quad V\,dp + C_p\,dT = 0$

$p\,dV = -C_v\,dT \quad -①$

$V\,dp = +C_p\,dT \quad -②$

devide: $\frac{②}{①} \quad \frac{V\,dp}{p\,dV} = \frac{C_p}{C_v} = \gamma \Rightarrow \frac{dp}{p} = -\gamma \frac{dV}{V}$

integrate: $\int \frac{dp}{p} = -\gamma \int \frac{dV}{V}$

$\ln p = -\gamma \ln V + \text{Const.}$

$\ln p + \ln V^{\gamma} = \text{Const} = \ln \text{Const}'$

$\ln (pV^{\gamma}) = \ln \text{Const}' \Rightarrow \boxed{pV^{\gamma} = \text{Const}}$