Chapter 16

A spring stretches by 4 cm when a 10-g mass is hung from it. If a total mass of 25 g attached to this spring oscillates in simple harmonic motion, calculate the period of the motion.

A. 0.635 sec
B. 0.401 sec
C. 0.854 sec
D. 1.000 sec
E. 0.250 sec

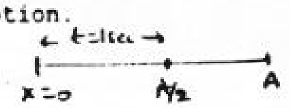
$$F = kx = mg \Rightarrow k = \frac{mg}{x} = 2.45\ \frac{N}{m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025}{2.45}} = \underline{0.635\ \text{sec}}$$

A small body is undergoing simple harmonic motion of amplitude A. While going to the right from the equilibrium position, it takes 1 second to move from x = + A/2 to x = + A. Find the period of motion.

A. 7.0 s
B. 8.0 s
C. 2.0 s
D. 4.0 s
E. 6.0 s

$$\frac{T}{8} = 1\ \text{sec} \Rightarrow T = 8\ \text{sec}$$



A body moves with simple harmonic motion. At t=0 it is released from rest at a displacement x=0.5 m. If the frequency of the oscillations is 5 Hz, find the the displacement x at t=0.02 s.

A. 0.3 m
B. 0.5 m
C. 0.4 m
D. 0.1 m
E. 0.2 m

$$f = 5\ Hz \rightarrow T = \frac{1}{f} = 0.2\ \text{sec}$$

$x_m = 0.5\ m$ and $\omega = 2\pi f = 10\pi$ rad/s

$$x = x_m \cos(\omega t + \phi)$$

$\phi = 0$ $\quad x = 0.5 \cos(10\pi t)$

$t = 0.02\ s \Rightarrow x = 0.4\ m$

An oscillatory mass-spring system has a mechanical energy of 1.0 J, an amplitude of 0.10 m and a maximum speed of 1.0 m/s. What is the mass ?

A. 1 kg
B. 7 kg
C. 2 kg
D. 5 kg
E. 6 kg

$$E = 1.0\,J \qquad A = 0.1\ m$$

$$v_{max} = 1.0\ m/s = \omega A \Rightarrow \omega = \frac{1.0}{0.1} = 10\ \frac{rad}{s}$$

$$E = \frac{1}{2} k A^2 \rightarrow k = \frac{2E}{A^2} = \frac{2}{(0.1)^2} = 200\ \frac{N}{m}$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{\omega^2} = \frac{200}{100} = 2\ kg$$

A 0.8 kg block attached to a spring oscillates with simple harmonic motion according to the equation x = 0.5 (m) * cos(20 (rad/s) * t (s)). What is the potential energy stored in the spring when the block's velocity is 5 m/s ?

A. 70 J
B. 50 J
C. 60 J
D. 30 J
E. 40 J

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} m v^2 + U$$

$$U = \frac{1}{2} k A^2 - \frac{1}{2} m v^2$$

$$\omega = 20\ \frac{rad}{s} \qquad k = \omega^2 m = 320\ \frac{N}{m}$$

$$U = \frac{1}{2}(320)(0.5)^2 - \frac{1}{2}(0.8)(5)^2 = 30\,J$$

A 2-kg body oscillates with simple harmonic motion according to the equation x = 6 * cos (3*pi*t + pi/3) where x is in meters, t is in seconds and the expression in the parentheses is in radians. What is the total energy of the system ?

A. 5.895 J
B. 3.198*(10**3) J
C. zero
D. 1000 J
E. 2.794*(10**3) J

$$E = \frac{1}{2} k A^2$$

$$k = \omega^2 m = (3\pi)^2 \times 2 = 177.6\ \frac{N}{m}$$

$$E = \frac{1}{2}(177.6) \times 6^2 = 3198\,J$$

A mass of 0.10 kg is connected to a spring and is free to oscillate on a horizontal frictionless surface. The mass is displaced 5.0 cm from the equilibrium position and released from rest. If the period of the motion is 0.60 s, find the force constant of the spring.

A. 11 N/m
B. 15 N/m
C. 28 N/m
D. 7.7 N/m
E. 9.5 N/m

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m$$

$$\omega = 2\pi f = \frac{2\pi}{T} = 10.5 \text{ rad/s}$$

$$k = (10.5)^2 \times 0.1 = \underline{\underline{11 \ \frac{N}{m}}}$$

A simple harmonic oscillator has an amplitude of 10.0 cm. For what value of the displacement is the kinetic energy of the oscillator equal to three times its potential energy?

A. 2.50 cm
B. 7.50 cm
C. 3.33 cm
D. 6.67 cm
E. 5.00 cm

$$K = 3U$$

$$\frac{1}{2} m v^2 = 3 \ \frac{1}{2} k x^2$$

$$E = \frac{1}{2} k A^2 = K + U = 3\left(\frac{1}{2} k x^2\right) + \frac{1}{2} k x^2$$

$$\frac{1}{2} \cancel{k} A^2 = 2 \cancel{k} x^2$$

$$x = \pm\sqrt{\frac{A^2}{4}} = \pm \frac{A}{2} = \underline{\underline{\pm 5 \text{ cm}}}$$

A 2.0 kg mass hangs on the end of a spring in the rest position. A force of 2.0 N pulls the mass down an additional 4.0 cm. If the force is then suddenly removed, the mass executes SHM. Find the total energy of the motion.

A. 2.2 J
B. 4.0 J
C. 0.26 J
D. 0.60 J
E. 0.04 J

$$E = \frac{1}{2} k A^2 \qquad F = kx$$

$$2 = k(0.04) \Rightarrow k = 50 \ \frac{N}{m}$$

$$E = \frac{1}{2}(50)(0.04)^2 = \underline{\underline{0.04 \text{ J}}}$$

A mass of 0.50 kg connected to a light spring of force constant 24 N/m oscillates on a horizontal frictionless surface with an amplitude of 4.0 cm. What is the speed of the mass when the displacement is equal to 2.0 cm ?

A. 72 cm/s
B. 12 cm/s
C. 36 cm/s
D. 24 cm/s
E. 48 cm/s

$m = 0.5\ kg \quad k = 24\ \frac{N}{m} \quad A = 4.0\ cm$

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$v = \sqrt{\frac{k}{m} A^2 - \frac{k}{m} x^2}$$

$$= 0.24\ m/s = \underline{\underline{24\ cm/s}}$$

A small mass on the end of an ideal spring is pulled vertically downward from its equilibrium position a distance of 5.0 cm and released from rest. The mass then oscillates in SHM with a period of 8.0 s. Find the maximum speed of the mass.

A. 2.0 cm/s
B. 10.0 cm/s
C. 4.5 cm/s
D. 2.8 cm/s
E. 3.9 cm/s

$T = 8.0\ s \qquad \omega = \frac{2\pi}{T} = 0.79\ \frac{rad}{s}$

$$v_{max} = \omega A = 0.039\ m$$

$$= \underline{3.9\ cm}$$

Consider an ideal mass-spring system executing a simple harmonic motion along the x-axis. Which of the following statements is CORRECT in this case?

A. The potential energy is maximum at the equilibrium position.
B. The period of oscillation is independent of the mass.
C. The kinectic energy is maximum at the position of maximum displacement from the equilibrium.
D. The acceleration of the system is a constant of motion.
E. The total energy of the system is a constant of motion.

A simple pendulum of length 1.00 m makes 100 complete oscillations in 204 seconds at a certain location. What is the acceleration due to gravity this point ?

A. 15.23 m/(s**2)
B. 2.98 m/(s**2)
C. 9.49 m/(s**2)
D. 7.94 m/(s**2)
E. 11.75 m/(s**2)

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

$$g = \frac{4\pi^2 \ell}{T^2}$$

$$T = \frac{204}{100} = 2.04\,s$$

$$g = \underline{\underline{9.49\ m/s^2}}$$

A simple pendulum having a period of 1 s on the surface of the earth is found to have a period of 2.46 s on the surface of the moon. What is the acceleration due to gravity on the surface of the moon ?

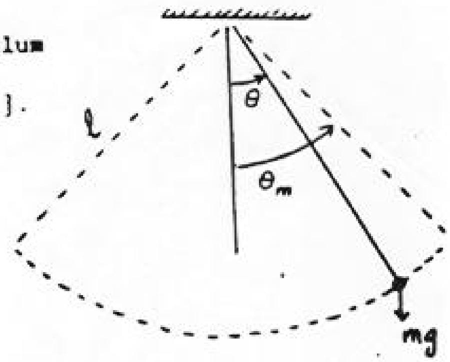
A. 3.71 m/s**2
B. 9.80 m/s**2
C. 16.10 m/s**2
D. 12.10 m/s**2
E. 1.62 m/s**2

$$T_e = 2\pi \sqrt{\frac{\ell}{g_e}} \qquad \frac{T_e}{T_m} = \sqrt{\frac{g_m}{g_e}}$$

$$T_m = 2\pi \sqrt{\frac{\ell}{g_m}} \qquad g_m = g_e \left(\frac{T_e}{T_m}\right)^2$$

$$g_m = 9.8\left(\frac{1}{2.46}\right)^2 = \underline{\underline{1.62\ m/s^2}}$$

The equation of motion for a certain simple pendulum pendulum (see figure) is
theta = (20 deg) * sin[(3*pi rad) * t + 0.6 rad].
Find the length of the pendulum.



$$\theta = 20^\circ \sin(3\pi t + 0.6)$$

A. 25 cm
B. 4.0 cm
C. 8.1 cm
D. 2.4 cm
E. 11 cm