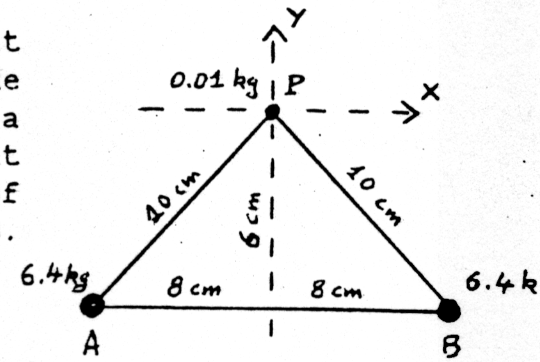
CHAPTER 14

Two spheres, each of mass 6.4 kg, are fixed at points A and B (see figure). Find the magnitude and direction of the initial acceleration of a sphere of mass 0.010 kg if released from rest at point P and acted only by forces of gravitational attraction of the spheres A and B.



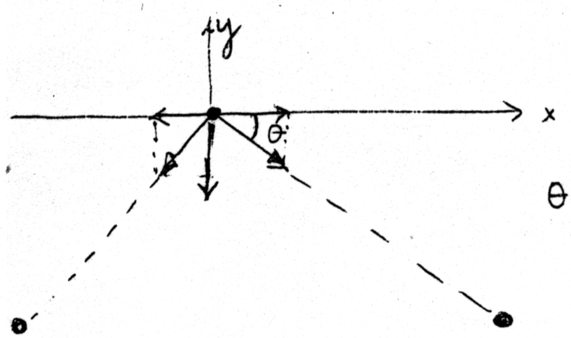
A. 0.51*10**(-7) m/s**2 (- $\vec{j}$)

B. 0.32*10**(-7) m/s**2 (- $\vec{j}$)

C. 0.11*10**(-6) m/s**2 (- $\vec{j}$)

D. 0.41*10**(-6) m/s**2 ($\vec{i}$ - $\vec{j}$)

E. 0.23*10**(-5) m/s**2 ($\vec{i}$ + $\vec{j}$)



$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 37^\circ$$

$$F_x = 0$$

$$F_y = \frac{2 G m_1 m_2}{r^2} \sin 37^\circ = 5.1 \times 10^{-10}\ \mathrm{N}$$

$$\vec{F} = 0\,\hat{i} - 5.1 \times 10^{-10}\ \mathrm{N}\ \hat{j}$$

$$\boxed{\vec{a} = \frac{\vec{F}}{m} = 0\,\hat{i} - 0.51 \times 10^{-7}\ \mathrm{m/s^2}\ \hat{j}}$$

A rocket is fired vertically from the earth's surface and reaches a maximum altitude above the surface of the earth equal to four earth radii. What is the initial speed of the rocket ?

A. 3.8 km/s
B. 10 km/s
C. 7.6 km/s
D. 16 km/s
E. 12 km/s

$$K_i + U_i = K_f + U_f$$

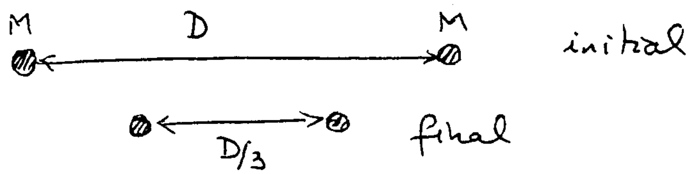
$$\frac{1}{2} m v_i^2 - \frac{GMm}{R_E} = \frac{1}{2} m v_f^2 - \frac{GMm}{5R_E}$$

$$v_i^2 = \cancel{v_f^2}^{\,0} + \frac{8}{5}\frac{GM}{R_E}$$

$$v_i = \sqrt{\frac{8}{5}\frac{GM}{R_E}} = 10\,000 \text{ m/s} = \underline{\underline{10 \text{ km/s}}}$$

Two particles of mass M are initially separated by a distance D. They are released from rest and accelerate towards one another through gravitational attraction. What is the kinetic energy of each particle when their separation distance is D/3 ? (G = gravitational constant)

A. 3*G*(M**2)/D
B. G*(M**2)/D
C. G*M/(2*(D**2))
D. 4*G*(M**2)/D
E. G*(M**2)/(2*D)



$$\cancel{K_i}^{\,0} + U_i = K_f + U_f$$

$$-\frac{GM^2}{D} = K_f - \frac{GM^2}{D/3} = K_f - \frac{3GM^2}{D}$$

$$\Rightarrow K_f = \frac{2GM^2}{D} \quad (\text{for both particles})$$

For one particle $\boxed{K_f = \frac{GM^2}{D}}$

A satellite is observed to orbit a large planet close to its surface with a period of 6.00 hours. Find the average mass density of the planet. Assume that the planet is spherical.

A. 2725 kg/(m**3)
B. 1.29 kg/(m**3)
C. 170 kg/(m**3)
D. 303 kg/(m**3)
E. 5522 kg/(m**3)

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T^2 = \frac{4}{3}\pi r^3 \left(\frac{3\pi}{GM}\right) = \left(\frac{V}{M}\right)\frac{3\pi}{G} = \frac{1}{\rho}\ \frac{3\pi}{G}$$

$$\Rightarrow \quad \rho = \frac{3\pi}{G T^2} = 303\ \text{kg/m}^3$$

Change to seconds

At what altitude (in earth's radii) above the surface of the earth would the acceleration of gravity be 1/8 of that on the surface ? (Re = radius of the earth)

A. 0.65 * Re
B. 1.83 * Re
C. 2.51 * Re
D. 1.02 * Re
E. 0.44 * Re

$$a_g = \frac{GM}{(R_E + h)^2} = \frac{1}{8}\ \frac{GM}{R_E^2}$$

$$\Rightarrow \quad (R_E + h)^2 = 8 R_E^2$$

$$R_E + h = \sqrt{8}\, R_E \Rightarrow \boxed{h = 1.83 R_E}$$

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A 100 kg spaceship is in circular orbit of radius 1.38 * 10**7 m around the earth. How much energy is required to transfer the spaceship to a circular orbit of radius 1.92 * 10**7 m ?

A. 9.51 * 10**9 J
B. 4.08 * 10**8 J
C. 3.42 * 10**8 J
D. 6.59 * 10**9 J
E. 6.72 * 10**8 J

$$\Delta E = E_f - E_i$$

$$= -\frac{GMm}{2r_f} + \frac{GMm}{2r_i}$$

$$= +\frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(100)}{2}\left(\frac{1}{1.38\times10^{7}} - \frac{1}{1.92\times10^{7}}\right)$$

$$\boxed{\Delta E = 4.08\times10^{8}\,\mathrm{J}}$$

A particle is at a height of 1000 km from the surface of the earth. Calculate the escape velocity of this particle. Assume the earth to be a perfect sphere of radius 6400 km and of mass 5.98*(10**24) kg.

A. 10.05 kilometers/second
B. 11.20 kilometers/second
C. 10.75 kilometers/second
D. 10.38 kilometers/second
E. 9.75 kilometers/second

$$U_i + K_i = \cancel{U_f}^{\,0} + \cancel{K_f}^{\,0}$$

$$-\frac{GMm}{r_i} + \frac{1}{2}m v_{esc}^2 = 0 \Rightarrow v_{esc} = \sqrt{\frac{2GM}{r_i}}$$

$$\boxed{v_{esc} = 10.4\times10^{3}\ \mathrm{m/s}}$$

The planet Mars has a satellite, Phobos , which travels in a circular orbit of radius 9.40*10**6 m , with a period of 2.754*10**4 s. Calculate the mass of Mars from this information.

A. 4.56*10**26 kg
B. 6.48*10**23 kg
C. 3.95*10**23 kg
D. 5.90*10**26 kg
E. Data incomplete. Mass of Phobos is not given.

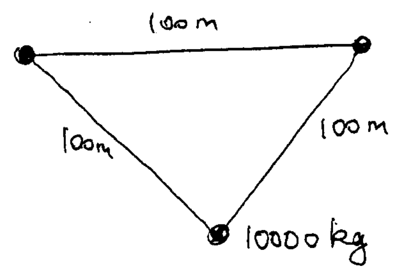
Use : $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

$$(2.754\times10^4)^2 = \left(\frac{4\pi^2}{6.67\times10^{-11} M}\right)(9.4\times10^6)^3$$

$$\Rightarrow \boxed{M = 6.48\times10^{23}\ kg}$$

Three particles each of mass 10000 kg each are placed at the corners of an equilateral triangle with each side 100 m long. Calculate the potential energy of the system.

A. -2.00*(10**-4) J
B. -4.50*(10**-4) J
C. -3.18*(10**-4) J
D. -6.97*(10**-4) J
E. -8.00*(10**-4) J

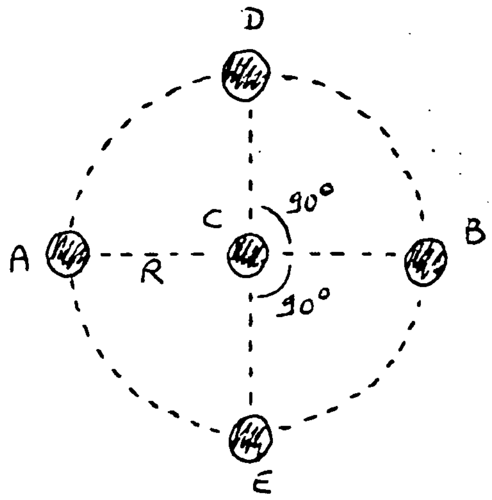


$$U = -\frac{G m_1 m_2}{r_{12}} - \frac{G m_1 m_3}{r_{13}} - \frac{G m_2 m_3}{r_{23}}$$

$$= -G\left(\frac{3m^2}{r}\right) = -6.67\times10^{-11}\left(3\times\frac{10000^2}{100}\right)$$

$$\boxed{U = -2\times10^{-4}\ J}$$

Four stars (A, B, D, E), of equal mass, rotate in the same direction around a fifth star C of the same mass located at their common center of mass (see figure). The radius of the common orbit is R. What minimum speed would star A need in order to depart from its companions for good? (express your answer in terms of G, M, R).



A. 1.23 * (G*M/R)**1/4
B. (G*M/R)**1/3
C. 5.32 * (G*M/R**3)**1/3
D. 2.41 * (G*M/R)**1/2
E. 3.21 * (G*M/R**2)**1/2

$$K_i + U_i = \underbrace{\cancel{K_f}^{\,0} + \cancel{U_f}^{\,0}}_{\text{depart for good}}$$

$$\frac{1}{2} m v_i^2 - G\left(\frac{m^2}{R} + \frac{m^2}{2R} + \frac{m^2}{R\sqrt{2}} + \frac{m^2}{R\sqrt{2}}\right) = 0$$

$$\frac{1}{2} m v_i^2 = \frac{Gm^2}{R}\left(1 + \frac{1}{2} + \frac{2}{\sqrt{2}}\right)$$

$$v_i = \sqrt{\frac{5.8\, Gm}{R}} = 2.41\sqrt{\frac{Gm}{R}}$$