Summary of chapter 9

Prepared by Dr. A. Mekki

- ***The center of mass of a system of particles is that point that moves as if <u>all the mass were concentrated there</u> and <u>all external forces were applied there</u>.***

- The coordinates of the center of mass in three dimensions are given by:

  (i) for a **<u>system of particles</u>**:

$$x_{cm} = \frac{1}{M}\sum m_i x_i \ ; \qquad y_{cm} = \frac{1}{M}\sum m_i y_i \ ; \qquad z_{cm} = \frac{1}{M}\sum m_i z_i$$

  (ii) for a **<u>rigid boby</u>**:

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm \qquad z_{cm} = \frac{1}{M}\int z\,dm$$

- If the system of particles is moving due to external forces, then Newton second law can be applied **<u>to the center of mass</u>** of the system as is given by:

$$\sum F_{ext} = Ma_{cm}$$

where ΣF_{ext} is the net external force, M is the total mass of the system and a_{cm} is the aceleration of the center of mass of the system.

Also ; $v_{cm} = \frac{1}{M}\sum v_i m_i$ where v_{cm} is the velocity of the center of mass of the system of particles.

<u>Linear momentum</u>

(i) for a single particle: $\vec{p} = m\vec{v}$ **,** where m is the mass and v is the velocity of the particle.

The unit of the linear momentum p is (kg m/s).

$\vec{p}$ and $\vec{v}$ have the same direction.

Newton second law defined in chapter 5 can also be written as

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

(ii) **for a system of particles:**
The total momentum of a system of particles is

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = M\vec{v}_{cm}$$

where p_1, p_2, p_3,   is the momentum of each particle.

and Newton second law is written as:

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

➢ Conservation of linear momentum

If a system is **isolated and closed** so that NO net external force act on the system, then ***the linear momentum of the system remains constant***.

$$\vec{P}_i = \vec{P}_f$$

In three dimensions:

$$\Rightarrow (P_i)_x = (P_f)_x$$
$$(P_i)_y = (P_f)_y$$
$$(P_i)_z = (P_f)_z$$

We say that the linear momentum is conserved.

A good example would be an explosion on a frictionless table, where the forces are internal and the linear momentum is conserved because the net external force is zero in this case.

Summary of chapter 10

Prepared by Dr. A. Mekki

- ***A collision is an event in which two or more bodies exert strong forces on each other for a relatively short time.***

Impulse and linear momentum:

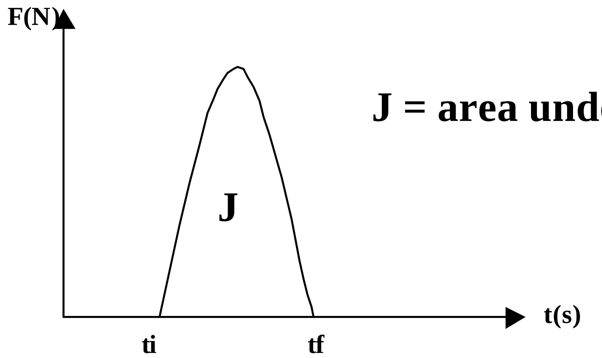
The impulse is defined as the change in linear momentum, that is:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

The force exerted by one body on the other body is:

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{J} = \Delta\vec{p} = \int_{t_f}^{t_i} \vec{F}dt$$

This force is represented graphically as:



J = area under the curve of F versus t graph.

If $\overline{F}$ is the average force during the collision time Δt, then the impulse is related to the force as:

- For a single particle projectile:

$$J = \overline{F}\Delta t \text{ or } \overline{F} = \frac{J}{\Delta t}$$

- For multiple particles projectile and one fixed target:

$$\overline{F} = -(\frac{n}{\Delta t})\Delta p$$

where $(n/\Delta t)$ is the rate at which the bodies collide with the fixed body and Δp is the change in momentum of each colliding body projectile.

➢ Elastic collision in one dimension:

A collision is said to be elastic if the kinetic energy before and after the collision is the same. In other words, kinetic energy is conserved. Here there are two situations:

- **Fixed target**:

  Here the target is stationary ($v_{2i}=0$) and the projectile is moving with velocity v_{1i}. From conservation of linear momentum and kinetic energy we get the final velocities as:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

- **Moving target:**

  Here both the target and the projectile are moving. Then the final velocities of the target and the projectile are:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Since there are no external forces during a collision, the velocity of the center of mass before and after the collision is the same and is given by:

$$v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

➢ Inelastic collision in one dimension:

In the case of inelastic collision the kinetic energy before and after the collision is not the same. However, since the system is closed and isolated, momentum is conserved. Kinetic energy is lost due to deformation of the bodies and heat.

If the colliding bodies stick together after the collision, the collision is said to be completely inelastic. In one-dimension we have conservation of momentum:

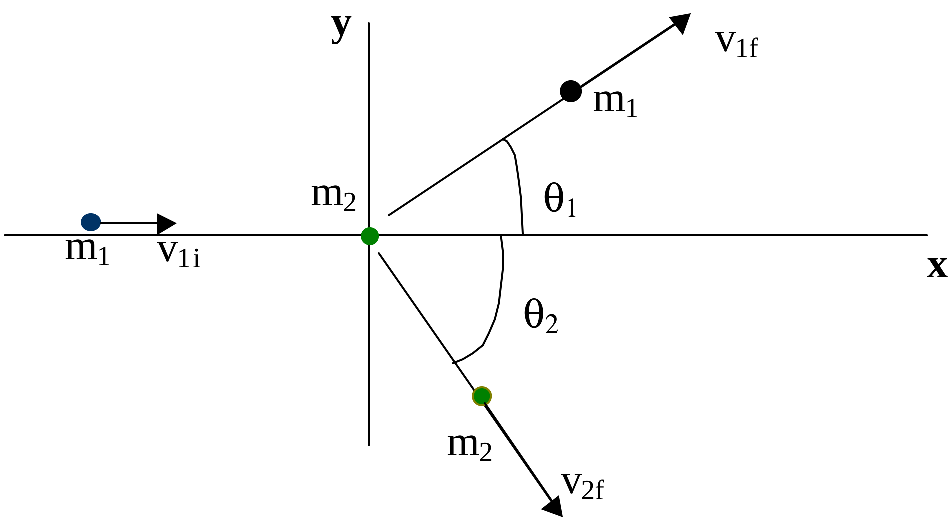
$$p_i = p_f \Rightarrow m_1 v_1 + m_2 v_2 = (m_1 + m_2)V$$

where V is the velocity of the two particles after the collision.

and $$v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = V$$

➢ Elastic collision in two dimension:

Here we consider a ***glancing collision*** between the projectile body and the target body. In this case the collision in **two-dimensional** according to the figure below:



Since the linear momentum is conserved, then:

x-axis: $$m_1 v_{1i} = m_1 v_{1f} \cos q_1 + m_2 v_{2f} \cos q_2$$

y-axis: $$0 = m_1 v_{1f} \sin q_1 - m_2 v_{2f} \sin q_2$$

If the kinetic energy is conserved (elastic collision), then:

$$\frac{1}{2} m v_{1i}^2 = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2$$

There are 7 variables is the above three equations. Generally, the unkowns are θ_1, θ_2, v_{1f} and v_{2f}. Generally one of these quantities is given or,

♦ If the two colliding masses are equal, the angles $\theta_1 + \theta_2 = 90^o$

➢ Reactions and decay processes:

Here we deal with a special type of collisions called nuclear reactions or decays. In both cases the linear momentum and total energy are conserved.

- Decay: $^{235}U \longrightarrow \alpha + {}^{231}Th$ (example 10.9 in the textbook)

  U: uranium, α is the alpha particle, and Th: thorium

- Nuclear reactions: $d + d \longrightarrow t + p$ (example 10.11 in the textbook)

  d: deuteron, t:triton, and p: proton.

We define the Q factor (mass energy) as: $Q = -(m_f - m_i)\,c^2 = -\Delta m\, c^2$
It has the unit of energy.

If Q is positive, then the reaction is **exothermic** (release of energy).
If Q is negative, then the reaction is **endothermic** (need of energy).