4E. The angular position of a point on the rim of a rotating wheel is given by $\theta = 4.0t - 3.0t^2 + t^3$, where θ is in radians and t is in seconds. What are the angular velocities at (a) $t = 2.0$ s and (b) $t = 4.0$ s? (c) What is the average angular acceleration for the time interval that begins at $t = 2.0$ s and ends at $t = 4.0$ s? What are the instantaneous angular accelerations at (d) the beginning and (e) the end of this time interval?

a) $\omega = \frac{d\theta}{dt} = 4 - 6t + 3t^2$

$t = 2s \quad \omega = 4 - 6\times 2 + 3\times(2)^2 = 4$ rad/s

b) $t = 4s \quad \omega = 4 - 6\times 4 + 3\times(4)^2 = 28$ rad/s

c) $\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{28-4}{4-2} = 12$ rad/s^2

d) $\alpha = \frac{d\omega}{dt} = -6 + 6t$

$t = 2s \quad \alpha = -6 + 12 = 6$ rad/s^2

$t = 4s \quad \alpha = -6 + 24 = 18$ rad/s^2

In this problem α is not constant

12E. Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?

$\alpha =$ Constant

$\omega_0 = 0$

$\Delta t = 5\text{ s} \quad \Delta\theta = 25\text{ rad}$

a) $\Delta\theta = \frac{1}{2}\alpha t^2 + \cancel{v_0 t}^{\,0} \Rightarrow 25 = \frac{1}{2}\alpha (5)^2$

$\Rightarrow \alpha = 2\text{ rad/s}^2$

b) $\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{25}{5} = 5\text{ rad/s}$

c) $\omega = \cancel{\omega_0}^{\,0} + \alpha t = 2 \times 5 = 10\text{ rad/s}$

d) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 10 \times 5 + \frac{1}{2} \times 2 \times (5)^2$

$= 50 + 25 = 75\text{ rad}$

28P. A gyroscope flywheel of radius 2.83 cm is accelerated from rest at 14.2 rad/s^2 until its angular speed is 2760 rev/min. (a) What is the tangential acceleration of a point on the rim of the flywheel during this spin-up process? (b) What is the radial acceleration of this point when the flywheel is spinning at full speed? (c) Through what distance does a point on the rim move during the spin-up?

$R = 2.83 \text{ cm} = 2.83 \times 10^{-2} \text{ m}$

$\omega_0 = 0 \qquad \alpha = 14.2 \text{ rad/s}^2$

$\omega = 2760 \dfrac{\text{rev}}{\text{min}} = 2760 \dfrac{2\pi}{60} \dfrac{\text{rad}}{\text{sec}} = 289 \text{ rad/s}$

a) $a_t = \alpha r = (14.2) \times (2.83 \times 10^{-2}) = 0.402 \text{ m/s}^2$

b) $a_r = \dfrac{v^2}{r} = \omega^2 r = (289)^2 \times (2.83 \times 10^{-2}) = 2.36 \times 10^3 \text{ m/s}^2$

c) $s = \theta r$

to find θ use $\omega^2 - \omega_0^2 = 2\alpha\theta$

$\theta = \dfrac{\omega^2 - \cancel{\omega_0^2}}{2\alpha} = \dfrac{(289)^2}{2 \times (14.2)} = 2940.1 \text{ rad}$

$\Rightarrow s = (2940.1) \times (2.83 \times 10^{-2}) = 83.2 \text{ m}$

37E. In Fig. 11-34, two particles, each with mass m, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length d and mass M. The combination rotates around the rotation axis with angular velocity ω. In terms of these symbols, and measured about O, what are the combination's (a) rotational inertia and (b) kinetic energy?

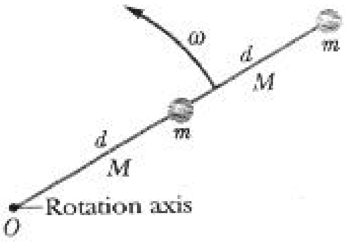


Fig. 11-34 Exercise 37.

a) $I_{Total} = I_1 + I_2 + I_3 + I_4$

$I_1 = m d^2$

$I_2 = m (2d)^2 = 4 m d^2$

$I_3 = \frac{1}{12} M d^2 + M \left(\frac{d}{2}\right)^2 = \frac{1}{3} M d^2$

$I_4 = \frac{1}{12} M d^2 + M \left(\frac{3d}{2}\right)^2 = \frac{7}{3} M d^2$

$I_{Total} = 5 m d^2 + \frac{8}{3} M d^2$

b) $K_{Total} = \frac{1}{2} I_{Total} \omega^2 = \frac{1}{2} \left(\frac{8}{3} M d^2 + 5 m d^2\right) \omega^2$

$K_{Total} = \left(\frac{4}{3} M d^2 + \frac{5}{2} m d^2\right) \omega^2$

48P. The body in Fig. 11-38 is pivoted at O. Three forces act on it in the directions shown: $F_A = 10$ N at point A, 8.0 m from O; $F_B = 16$ N at point B, 4.0 m from O; and $F_C = 19$ N at point C, 3.0 m from O. What is the net torque about O?

Fig. 11-38 Problem 48.

$$\tau_{net} = \tau_A + \tau_B + \tau_C$$

$$= F_A r_A \sin\theta_A - F_B r_B \sin\theta_B + F_C r_C \sin\theta_C$$

$$= 10 \times 8 \times \sin 45° - 16 \times 4 \times \sin 90° + 19 \times 3 \times \sin 20°$$

$$\tau_{net} = 12.1 \text{ N.m}$$

It is positive $\Rightarrow$ the object will rotate counterclockwise

52E. In Fig. 11-39, a cylinder having a mass of 2.0 kg can rotate about its central axis through point O. Forces are applied as shown: $F_1 = 6.0$ N, $F_2 = 4.0$ N, $F_3 = 2.0$ N, and $F_4 = 5.0$ N. Also, $R_1 = 5.0$ cm and $R_2 = 12$ cm. Find the magnitude and direction of the angular acceleration of the cylinder. (During the rotation, the forces maintain their same angles relative to the cylinder.)

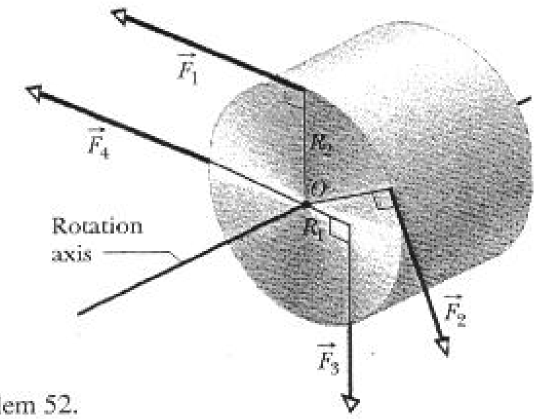


Fig. 11-39 Problem 52.

Use Newton's Second Law $\tau_{net} = I\alpha$

$$\Rightarrow \alpha = \frac{\tau_{net}}{I}$$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2)(0.12)^2 = 0.0144 \text{ Kg}\cdot\text{m}^2$$

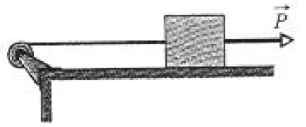
$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \cancel{\tau_4}^{\,0}$$

$$= F_1 R_2 - F_2 R_2 - F_3 R_1$$

$$= 6 \times (0.12) - 4 \times (0.12) - 2 \times (0.05) = 0.14 \text{ N}\cdot\text{m}$$

$$\Rightarrow \alpha = \frac{0.14}{0.0144} = 9.7 \text{ rad/s}^2$$

54P. A wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.050 kg · m^2. A massless cord wrapped around the wheel is attached to a 2.0 kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude $P = 3.0$ N is applied to the block as shown in Fig. 11-41, what is the magnitude of the angular acceleration of the wheel? Assume the string does not slip on the wheel.

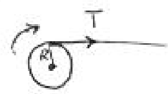


$R = 0.20$ m $\qquad P = 3.0$ N

$I = 0.05$ Kg·m^2 $\qquad \alpha = ?$

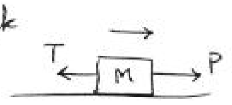
$m = 2.0$ Kg

Apply Newton's second law to the wheel



$$TR = I\alpha \quad \text{—— (1)}$$

Apply Newton's second law to the block



$$P - T = Ma$$

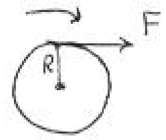
but $a = R\alpha \Rightarrow P - T = MR\alpha \quad \text{—— (2)}$

$$P - \frac{I\alpha}{R} = MR\alpha \Rightarrow P = \left(\frac{I}{R} + MR\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{P}{\frac{I}{R} + MR} = \frac{3}{\frac{0.05}{0.2} + 2 \times 0.2} = \frac{3}{0.65}$$

$\alpha = 4.6$ rad/s^2

56P. A pulley, with a rotational inertia of 1.0×10^{-3} kg·m^2 about its axle and a radius of 10 cm, is acted on by a force applied tangentially at its rim. The force magnitude varies in time as $F = 0.50t + 0.30t^2$, with F in newtons and t in seconds. The pulley is initially at rest. At $t = 3.0$ s what are (a) its angular acceleration and (b) its angular speed?



a) $\tau = I\alpha = FR$

$$\Rightarrow \alpha = \frac{FR}{I}$$

at $\underline{t = 3s}$ $\qquad F = 0.5 \times 3 + 0.3 \times (3)^2 = 4.2\ \text{N}$

$$\alpha = \frac{4.2 \times 0.1}{1 \times 10^{-3}} = 420\ \text{rad/s}^2$$

b) $\omega = \omega_0 + \alpha t$ cannot be used because α is <u>not constant</u> in this problem

$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega = \int_0^3 \alpha\, dt$$

$$\omega = \int_0^3 (50t + 30t^2)\, dt = 25t^2 + 10t^3 \Big|_0^3$$

$$= 495\ \text{rad/s}$$

60E. A 32.0 kg wheel, essentially a thin hoop with radius 1.20 m, is rotating at 280 rev/min. It must be brought to a stop in 15.0 s. (a) How much work must be done to stop it? (b) What is the required average power?

Thin hoop $I = MR^2$ (see table 11.2 in your textbook)

$\omega_o = 280 \frac{rev}{min} = 280 \times \frac{2\pi \text{ rad}}{60 \text{ s}} = 29.3 \text{ rad/s}$

$\omega_f = 0$

$t = 15$ sec

a) $W = \Delta K = K_f - K_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

$= 0 - \frac{1}{2} MR^2 \omega_i^2 = -\frac{1}{2}(32)(1.2)^2 \times (29.3)^2$

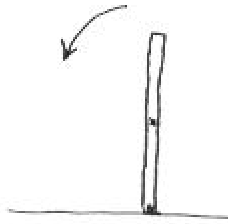
$= -19.8$ kJ

Work done is 19.8 kJ

b) $\bar{P} = \frac{W}{\Delta t} = \frac{19.8 \text{ kJ}}{15} = 1.32 \text{ kW}$

63P. A meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end when it hits the floor, assuming that the end on the floor does not slip. (*Hint:* Consider the stick to be a thin rod and use the conservation of energy principle.) ssm ilw

Use conservation of total energy



initial

$$\Delta K + \Delta U_g = 0$$

$$\left(\frac{1}{2} I \omega_f^2 - \cancel{\frac{1}{2} I \omega_i^2}\right) + \left(0 - mg\frac{\ell}{2}\right) = 0$$

$$\frac{1}{2} I \omega_f^2 = mg\frac{\ell}{2}$$

final

$$\omega_f^2 = \frac{mg\ell}{I} \quad \text{but } I = \frac{1}{3} m\ell^2$$

$$\omega_f^2 = \frac{\cancel{m}g\cancel{\ell}}{\frac{1}{3}\cancel{m}\ell^{\cancel{2}}} = \frac{3g}{\ell} \Rightarrow \omega_f = \sqrt{\frac{3g}{\ell}}$$

$$v_f = \omega_f \ell = \sqrt{3g\ell}$$

$$\ell = 1 \text{ m} \Rightarrow v_f = 5.4 \text{ m/s}$$