FORMULA SHEET – PHYS 301

Chapter 5

$$\mathbf{F} = -G\frac{mM}{r^2}\mathbf{e_r}, \quad G = 6.67\times10^{-11} N.m^2/kg^2$$

$$\mathbf{F} = -Gm\int_V \frac{\rho(\mathbf{r}')\mathbf{e_r}}{r^2}dV', \quad \mathbf{g} = -G\int_V \frac{\rho(\mathbf{r}')\mathbf{e_r}}{r^2}dV'$$

$$\mathbf{g} = -\nabla\Phi, \quad \Phi = -G\int_V \frac{\rho(\mathbf{r}')}{r}dV'$$

Chapter 7

$$\delta\int_{t_1}^{t_2} L(q_j,\dot{q}_j,t)dt = 0$$

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} = 0, \qquad j = 1,2,...,s$$

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \sum_k \lambda_k(t)\frac{\partial f}{\partial q_j} = 0, \qquad f(q_j,\dot{q}_j,t) = 0$$

$$p_j = \frac{\partial L}{\partial \dot{q}_j}, \quad \dot{p}_j = \frac{\partial L}{\partial q_j}$$

$$H(q_k,p_k,t) = \sum_j p_j\dot{q}_j - L(q_k,\dot{q}_k,t)$$

$$\dot{q}_k = \frac{\partial H}{\partial p_k}, \qquad -\dot{p}_k = \frac{\partial H}{\partial q_k} \qquad \langle T\rangle = \frac{n+1}{2}\langle U\rangle$$

Chapter 8

$$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$$

$$\ell = \mu r^2\dot{\theta} = \text{constant} \quad E = \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\frac{\ell^2}{\mu r^2} + U(r)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2}F(r) \quad V(r) = U(r) + \frac{\ell^2}{2\mu r^2}$$

$$\frac{\alpha}{r} = 1 + \varepsilon\cos\theta \quad \alpha = \frac{l^2}{\mu k} \quad \varepsilon = \sqrt{1 + \frac{2El^2}{\mu k^2}} \quad a = \frac{\alpha}{1-\varepsilon^2} \quad b = \sqrt{\alpha a}$$

$$r_{\min} = a(1-\varepsilon) \quad r_{\max} = a(1+\varepsilon) \quad \tau^2 = \frac{4\pi^2}{GM}a^3$$

$G = 6.67\text{x}10^{-11}$ Nm^2/kg^2
$M_E = 5.92\text{x}10^{24}$ kg
$R_E = 6371$ km

Chapter 6

$$J = \int_{x_1}^{x_2} f\{y_i(x), y_i'(x); x\}dx, \quad i = 1,2,...,n$$

$$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y_i'} = 0, \quad i = 1,2,...,n$$

$$f - y'\frac{\partial f}{\partial y'} = \text{constant} \quad (\textit{for} \ \frac{\partial f}{\partial x} = 0)$$

$$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y_i'} + \sum_j \lambda(x)\frac{\partial g_j}{\partial y_i} = 0, \quad g_j\{y_i(x); x\} = 0$$

Chapter 9

$$R = \frac{1}{M}\sum_i m_i r_i \quad R = \frac{1}{M}\int r\,dm \quad \vec{P} = M\dot{\vec{R}} \quad \vec{L} = \vec{R}\times\vec{P} + \sum_i \vec{r}_i\,'\times\vec{p}_i\,'$$

$$T = \sum_i \frac{1}{2}m_i v_i\,'^2 + \frac{1}{2}MV^2 \quad u_1\,' = v_1\,' \quad u_2\,' = v_2\,'$$

$$\tan\psi = \frac{\sin\theta}{\cos\theta + (\frac{m_1}{m_2})} \quad \tan\zeta = \frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2} = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$T_0\,' = \frac{m_2}{m_1+m_2}T_0 \qquad T_1\,' = \left(\frac{m_2}{m_1+m_2}\right)^2 T_0 \qquad T_2\,' = \frac{m_1m_2}{(m_1+m_2)^2}T_0$$

$$\frac{T_1}{T_0} = 1 - \frac{2m_1m_2}{(m_1+m_2)^2}(1-\cos\theta) = \frac{m_1^{\,2}}{(m_1+m_2)^2}\left[\cos\psi \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\psi}\right]^2$$

$$\frac{T_2}{T_0} = \frac{4m_1m_2}{(m_1+m_2)^2}\cos^2\zeta \quad \frac{T_1}{T_0} = \cos^2\psi \qquad \frac{T_2}{T_0} = \sin^2\psi$$

$$Q + \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\varepsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|}$$

Chapter 12

$$\mathrm{A}_{jk} = \frac{\partial^2 U}{\partial q_j \partial q_k}, \ \mathrm{m}_{jk} = \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_k}$$

$$T = \frac{1}{2}\sum_{j,k} m_{jk}\dot{q}_j\dot{q}_k, \ U = \frac{1}{2}\sum_{j,k} A_{jk}\dot{q}_j\dot{q}_k$$

$$\sum_j \left(A_{jk} - \omega^2 m_{jk}\right)a_j = 0, \ \left|A_{jk} - \omega^2 m_{jk}\right| = 0$$

$$q_j(t) = \sum_r a_{jr}\eta_r(t)$$

Useful formulas

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right), \quad \int \frac{x dx}{a^2 - x^2} = -\frac{1}{2} \ln(a^2 - x^2)$$

$$\int \frac{dx}{a^2 x^2 - b^2} = \frac{1}{2ab} \ln\left(\frac{ax - b}{ax + b}\right)$$

$$\int e^{ax} \sin(x) dx = \frac{e^{ax}}{a^2 + 1}(a \sin x - \cos x)$$

$$\int e^{ax} \sin^2(x) dx = \frac{e^{ax}}{a^2 + 4}(a \sin^2 x - 2 \sin x \cos x + \frac{2}{a})$$