King Fahd University of Petroleum & Minerals
Physics Department
Phys212- Quiz#8

Name: Key ID#:

The electron in a hydrogen atom is in a state described by the wavefunction $\Psi_{322}(\mathbf{r})$.

(a) What is the magnitude of the orbital angular momentum of the electron in this state?

$$\psi_{n\ell m_\ell} \Rightarrow \ell = 2 \qquad |\vec{L}| = \hbar\sqrt{\ell(\ell+1)} = \boxed{\hbar\sqrt{6}}$$

(b) What is the angle between the angular momentum vector and the z-axis?

$$m_\ell = 2 \qquad \theta = \cos^{-1}\left(\frac{m_\ell}{\sqrt{\ell(\ell+1)}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) = \boxed{35.3^\circ}$$

(c) Calculate the average value of r for the electron in this state.
Given $\int_0^\infty x^n e^{-x} dx = n!$ $a_0 = 0.053$ nm.

$$\langle r \rangle = \int_0^\infty r\, r^2 |R_{32}(r)|^2 dr = \int_0^\infty r^3 \frac{1}{(3a_0)^3} \frac{4\times 2}{(27)^2\times 5} \left(\frac{r}{a_0}\right)^4 e^{-\frac{2r}{3a_0}} dr$$

$$\langle r \rangle = \frac{8}{98415} \frac{1}{a_0^7} \int_0^\infty r^7 e^{-\frac{2r}{3a_0}} dr$$

$$\text{let } z = \frac{2r}{3a_0} \Rightarrow r = \frac{3a_0 z}{2} \qquad dr = \frac{3}{2} a_0\, dz$$

$$\langle r \rangle = \frac{8}{98415} \frac{1}{a_0^7} \left(\frac{3a_0}{2}\right)^8 \underbrace{\int_0^\infty z^7 e^{-z} dz}_{7!} = \frac{8}{98415} \left(\frac{3}{2}\right)^8 7!\, a_0$$

$$\langle r \rangle = \frac{52488 \times 5040}{98415 \times 256} a_0 = \boxed{10.5\, a_0}$$