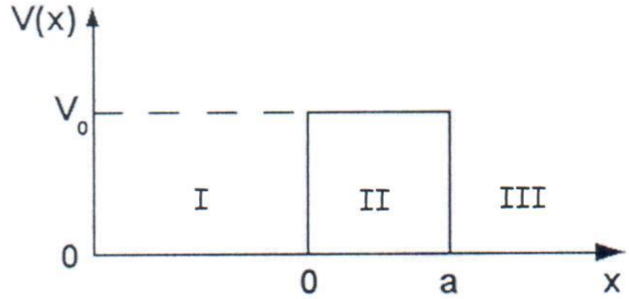
King Fahd University of Petroleum & Minerals
Physics Department
Phys212- Quiz#7

Name: Key ID#:

Consider a particle incident from the left on the square barrier shown in the figure. The particle has an energy $E < V_0$



(a) Write the time independent Schrodinger equation in regions I, II, and III.
(b) Write the expressions for the matter waves for the same three regions.
(c) Write the equations for the boundary conditions.

(a) I : $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx} = E\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} + \left(\frac{2mE}{\hbar^2}\right)\psi(x) = 0$

$\Rightarrow \frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$ where $k = \frac{\sqrt{2mE}}{\hbar}$

II : $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi(x) = E\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi(x) = 0$

$\Rightarrow \frac{d^2\psi}{dx^2} - \alpha^2\psi(x) = 0$ where $\alpha = \frac{\sqrt{2m(V_0-E)}}{\hbar}$

III : $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$ $k = \frac{\sqrt{2mE}}{\hbar}$

(b) I : $\psi_I(x) = Ae^{ikx} + Be^{-ikx}$

$\psi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$

$\psi_{III}(x) = Fe^{ikx} + \cancel{Ge^{-ikx}} \to 0$

(c) $\psi_I(0) = \psi_{II}(0) \Rightarrow A + B = C + D$ —— (1)

$\psi'_I(0) = \psi'_{II}(0) \Rightarrow ikA - ikB = \alpha C - \alpha D$ —— (2)

$\psi_{II}(a) = \psi_{III}(a) \Rightarrow Ce^{\alpha a} + De^{-\alpha a} = Fe^{ika}$ —— (3)

$\psi'_{II}(a) = \psi'_{III}(a) \Rightarrow \alpha Ce^{\alpha a} - \alpha De^{-\alpha a} = ikFe^{ika}$ —— (4)