# Chapter 3 Vectors

After reading this chapter the student should be able to:

1. Distinguish between **a** **scalar** and **a vector.**
2. Add vectors **geometrically** and **analytically**.
3. Be familiar with the **unit vector** notation.
4. Know how to perform **vector** (or **cross) product** and **scalar** (or **dot) product**.

## 3.1 Vectors and Scalars

Avector has a **magnitude** and a **direction**. Some physical quantities that are vector quantities are displacement, velocity, acceleration and force.

A scalar on the other hand is given as a **single value** with a sign. There are many scalar quantities in physics such as temperature, pressure, and mass.

A graphical representation of a vector quantity in two dimensions is shown below. Its length represents the magnitude of the physical quantity and the

arrow indicates the direction. A vector has a tail and a head.

Tail

Head

If we translate a vector without changing its magnitude and direction, the vector remains the same.

Translation 1

Translation 2

These three vectors are equivalents, i.e., they represent the same vector.

Suppose an object moves along the three paths as shown in the figure below. These three paths have the same displacement vector as they start at end at the

same points A and B.

**A**

**B**

Note that the distance traveled by the object in moving from A to B is different for the three paths. It is the biggest for the blue path.

## 3.2 Adding Vectors Geometrically

To add two vectors, say displacement vectors, means to evaluate the net displacement we can use what is called the **graphical method**. The **net** (or **resultant**)

displacement of two displacement vectors and is given by the vector equation. The resultant is also a vector. The procedure to add the vectors

geometrically is to bring the tail of at the head of keeping the orientation of the two vectors unchanged. The vector sum extends from the tail of vector to the head of vector.







Next, let us add three vectors, , and . The graphical way to do it is to perform the sum of two vectors say + and then add the third one. The vector equation is (+)+.

This sum is illustrated in the following diagram.























It is to be noted that . The law is associative.

The vector has the same magnitude as , but points in the opposite direction.

So .















This is the rule of vector subtraction.

If a vector is moved from one side of an equation to the other, a change is sign is needed similar to rules of algebra.

It is to be noted that only vectors of **the same kind** can be added or subtracted. We cannot add a displacement vector to a velocity vector!

## 3.3 Components of Vectors

Adding vectors geometrically does not tell us how to do the math with vector addition. We need to learn how to calculate vector quantities mathematically.

We will do this just in two dimensions, but it can be extended to three dimensions.

A **component** of a vector is the projection of the vector on one of the axes of a *set of cartesian coordinates* *system* as shown in the figure below.

*y*

*x*

*O*







**

***ax*** is the component of vector along the **x-axis** (or ***x-component***) and ***ay*** is the component of vector along the **y-axis** (or ***y-component***).

We say that the **vector is resolved** into its components.  is the angle the vector makes with the positive *x*-axis. In this figure both components of vector are positive.

Note that , graphically

*ax*

*ay*



**

The values of the components *ax* and *ay* can be found if the angle and the magnitude of the vector are known:



On the other hand, if the components *ax* and *ay* of the vector are known, we can find its magnitude and direction:

(using Pythagoras theorem) and 

General rule for the **sign** of the vector components:

Quadrant I

Quadrant II

Quadrant III

Quadrant IV

***x-component > 0***

***y-component > 0***

***x-component < 0***

***y-component > 0***

***x-component > 0***

***y-component <*** ***0***

***x-component < 0***

***y-component < 0***

## 3.4 Unit Vectors

A **unit vector** is one that has a magnitude of 1 and points in a particular direction. It is often indicated by putting a hat of top of the vector symbol, for example **unit vector** = and = 1.

We will label the unit vectors in the positive directions of the x, y , and z-axes as .

*x*

*y*

*z*







A two dimensional vector can be expressed in terms of unit vectors as . The quantities and are vectors and called

the **vector components** of while *ax* and *ay* are scalars and called the **scalar components** of .

## 3.5 Adding Vectors by Components

We have seen in section 3.2 how to add vectors graphically. The resolution of a vector into its components can be used to add and subtract vectors arithmetically.

To illustrate this let us take an example. What is the sum of the following three vectors using the components method?







The vector sum is 

The components of the vector are:



## 3.6 Multiplying Vectors

The multiplication of a vector by a scalar *m* gives a new vector.



If *m* > 0, the new vector will have the same direction as the vector .

If *m* < 0, the new vector will have apposite direction to the vector .

To divide by *m*, we multiply by .

There are two types of vector multiplication, namely, the **scalar** or **dot product** of two vectors, which results in a scalar,

and the **vector** or **cross product** of two vectors, which results in a vector.

The **scalar product** of two vectors and , denoted , is defined to be









and are the magnitudes of vector and vector, respectively.

And in particular we have , since the angle between a vector and itself is 0 and the cosine of 0 is 1.

Alternatively, we have , since the angle between  and  , and , and and is 90 and the cosine of 90 is 0.

The laws for scalar products are given in the following:



The **vector product** of two vectors and , denoted , produces a third vector whose magnitude is



**







The direction of is perpendicular to the plane that contains the two vectors and as shown in the figure.

In particular we have , since the angle between a vector and itself is 0 and the cosine of 0 is 1.

Note that the angle between  and  , and , and and is 90 and the sine of 90 is 1.

Therefore we use the following diagram to help us solve problems dealing with the cross product.













However, if the rotation in the figure is clockwise such as etc