

Quiz #2 Ch.#3 T133 Phys101.02-v1

Student ID:..... Student Name:.....

Q1. Q#7: If vector $A = 28\hat{i} + 11\hat{j}$ and vector B (magnitude of $B = 25$) as shown in figure 2, what is the magnitude of the sum of these two vectors? (Ans: 32)

$$B_x = B \cos 130 = 25 \cos 130 = -16.07$$

$$B_y = B \sin 130 = 25 \sin 130 = 19.15$$

$$\vec{C} = \vec{A} + \vec{B} = 28\hat{i} + 11\hat{j} + (-16.07\hat{i} + 19.15\hat{j})$$

$$\vec{C} = 11.93\hat{i} + 30.15\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} = 28\hat{i} + 11\hat{j} - 16.07\hat{i} + 19.15\hat{j}$$

$$\vec{C} = 11.93\hat{i} + 30.15\hat{j}$$

$$|\vec{C}| = \sqrt{(11.93)^2 + (30.15)^2} = 32.4$$

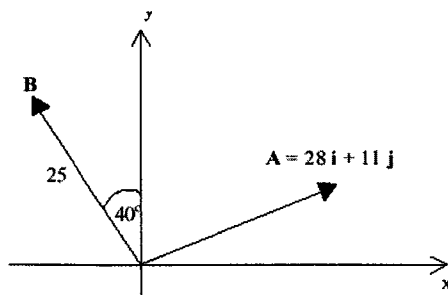


Figure 2

Q#2. Three vectors \vec{F} , \vec{v} and \vec{B} are related through $\vec{F} = 5.0(\vec{v} \times \vec{B})$. If vector $\vec{v} = 3.0\hat{i} - 5.0\hat{j}$ and $\vec{B} = -2.0\hat{k}$, then vector \vec{F} is: (Ans: $50\hat{i} + 30\hat{j}$)

$$\vec{F} = 5 \left[(3\hat{i} - 5\hat{j}) \times (-2\hat{k}) \right]$$

$$= 5 \left[(3\hat{i}) \times (-2\hat{k}) + (5\hat{j}) \times (2\hat{k}) \right]$$

$$= 30(\hat{i} \times \hat{k}) + 10(\hat{j} \times \hat{k}) = -30\hat{j} + 10\hat{i}$$

$$= 5 \left[-6(\hat{i} \times \hat{k}) + 10(\hat{j} \times \hat{k}) \right]$$

$$= 5 \left[6\hat{j} + 10\hat{i} \right] = 50\hat{i} + 30\hat{j}$$

Quiz #2 Ch.#3 T133 Phys101.02-v2

Student ID:..... Student Name:.....

Q#1. Vectors \vec{a} , \vec{b} , and \vec{c} are related through equations $\vec{a} + \vec{b} = \vec{c}$ and $\vec{a} - \vec{b} = 5.0\vec{c}$. If $\vec{c} = 3.0\hat{i} + 4.0\hat{j}$, what is the magnitude of vector \vec{a} ? (Ans: 15)

$$\begin{aligned}\vec{a} + \vec{b} &= \vec{c} \\ \vec{a} - \vec{b} &= 5.0\vec{c}\end{aligned}$$

$$2\vec{a} = 6.0\vec{c}$$

$$|\vec{a}| = \frac{3.0}{2} |\vec{c}| = 3 |\vec{c}| = 3 \times \sqrt{3^2 + 4^2} = 3 \times 5 = 15$$

Q#2.

You are given the two vectors \vec{A} and \vec{B} . If $\vec{A} \cdot \vec{B} = 4.00$ and $|\vec{A} \times \vec{B}| = 3.00$, then the angle between \vec{A} and \vec{B} is:

A) 36.9

$$A \cdot B = |A| |B| \cos \theta$$

$$A \times B = |A| |B| \sin \theta$$

$$\frac{A \times B}{A \cdot B} = \tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$$

Quiz #2 Ch.#3 T133 Phys101.02-v3

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Q1. Two vectors A and B are shown in Fig 1. Each vector has a magnitude of 5.0 m. Find the magnitude of the resultant vector $R = A + B$ and the angle (theta) between R and the positive x-axis (counter clockwise). (Ans: magnitude = 7.1 m, theta = 90 degrees)

$$\begin{aligned}
 A_x &= 5 \cos 45 = 3.53 \\
 A_y &= 5 \sin 45 = 3.53 \\
 B_x &= 5 \cos 135 = -3.53 \\
 B_y &= 5 \sin 135 = +3.53 \\
 \vec{R} &= \vec{A} + \vec{B} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} \\
 &= (3.53 + (-3.53))\vec{i} + (3.53 + 3.53)\vec{j} \\
 \vec{R} &= 7.06\vec{j} = 7.1\vec{j}
 \end{aligned}$$

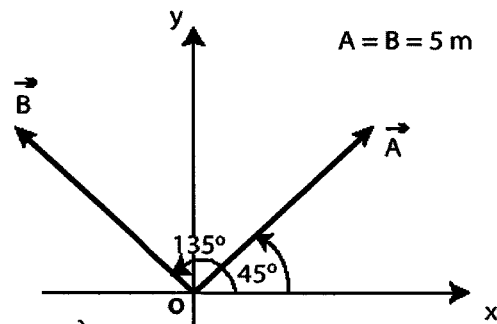


Figure 1

Q2. Consider two vectors \vec{A} and \vec{B} with magnitudes 5 cm and 8 cm, respectively. Vector \vec{A} is along the positive x-axis and vector \vec{B} is along the positive y-axis. Find $\vec{A} \cdot (\vec{A} + \vec{B})$. (Ans: 25 cm²)

$$\begin{aligned}
 \vec{A} &= 5\vec{i}, \quad \vec{B} = 8\vec{j} \\
 \vec{A} + \vec{B} &= 5\vec{i} + 8\vec{j} \\
 \vec{A} \cdot (\vec{A} + \vec{B}) &= 5\vec{i} \cdot (5\vec{i} + 8\vec{j}) = 25 \text{ cm}^2
 \end{aligned}$$

Quiz #2 Ch.#3 T133 Phys101.02-v4

Student ID:..... Student Name:.....

Q1. Vectors \vec{a} , \vec{b} , and \vec{c} are related through equations $\vec{a} + \vec{b} = \vec{c}$ and $\vec{a} - \vec{b} = 5.0\vec{c}$. If $\vec{c} = 3.0\hat{i} + 4.0\hat{j}$, what is the magnitude of vector \vec{a} ? (Ans: 15)

$$\vec{a} + \vec{b} = \vec{c}$$

Q2. For the following three vectors; $A=2i+3j+4k$, $B=4i+4j$ and $C= 2i+2k$, find $A.(B \times A)$. (Ans: 0.)

$$\vec{A} \cdot (\vec{B} \times \vec{A}) = (3\vec{i} + 3\vec{j} + 4\vec{k}) \cdot ((4\vec{i} + 4\vec{j}) \times (2\vec{i} + 2\vec{k}))$$

$$\vec{D} = (4\vec{i} + 4\vec{j}) \times (2\vec{i} + 2\vec{k}) = 8(\vec{i} \times \vec{i}) + 8(\vec{i} \times \vec{k}) + 8(\vec{j} \times \vec{i}) + 8(\vec{j} \times \vec{k})$$

$$\vec{D} = -8\vec{j} - 8\vec{k} + 8\vec{i} = 8\vec{i} - 8\vec{j} - 8\vec{k}$$

$$\vec{A} \cdot \vec{D} = (3\vec{i} + 3\vec{j} + 4\vec{k}) \cdot (8\vec{i} - 8\vec{j} - 8\vec{k})$$

~~$$= 24 - 24 - 32 = -32$$

$$\vec{D} = \vec{B} \times \vec{A} = (4\vec{i} + 4\vec{j}) \times (3\vec{i} + 3\vec{j} + 4\vec{k}) = 12(\vec{i} \times \vec{i}) + 12(\vec{i} \times \vec{j}) + 16(\vec{i} \times \vec{k}) + 12(\vec{j} \times \vec{i}) + 12(\vec{j} \times \vec{j}) + 16(\vec{j} \times \vec{k})$$

$$= 12\vec{k} - 16\vec{j} + 12\vec{k} + 16\vec{i} = 16\vec{i} + 24\vec{k} - 16\vec{j}$$

$$\vec{A} \cdot \vec{D} = (3\vec{i} + 3\vec{j} + 4\vec{k}) \cdot (16\vec{i} - 16\vec{j} + 24\vec{k}) = 48$$~~

Quiz #2 Ch.#3 T133 Phys101.02-v5

Student ID:..... Student Name:.....

Q#1: As shown in Fig. 3, a block moves down on a 45-degree inclined plane of 2.5 m length, then horizontally for another 2.5 m, and then falls down vertically a height of 2.5 m. Find the magnitude and direction of the resultant displacement vector of the block. (Ans: 6.0 m and 45 degrees below horizontal axis)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

$$= +2.5 \cos 225^\circ \hat{i} + 2.5 \sin 225^\circ \hat{j} - 2.5 \hat{i} - 2.5 \hat{j}$$

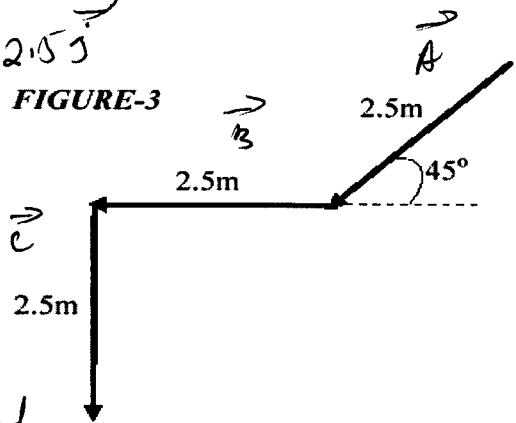
$$= -1.77 \hat{i} - 1.77 \hat{j} - 2.5 \hat{i} - 2.5 \hat{j}$$

$$= -4.27 \hat{i} - 4.27 \hat{j}$$

$$|\vec{D}| = \sqrt{(-4.27)^2 + (-4.27)^2} = 6.04 \text{ m}$$

$$\theta = \tan^{-1} \left(\frac{-4.27}{-4.27} \right) = 45^\circ \text{ in third quadrant}$$

=



Q2. Vectors \vec{F} and \vec{G} are defined as $\vec{F} = 3.0\hat{i} + 4.0\hat{j}$, and $\vec{G} = -\hat{i} + \hat{j}$. Find the component (projection) of vector \vec{G} along the direction of vector \vec{F} . (Ans: 0.20)

Component of \vec{G} along $\vec{F} = |\vec{G}| \cos \theta$

$$|\vec{F}| = \sqrt{3^2 + 4^2} = 5, \quad |\vec{G}| = \sqrt{1+1} = \sqrt{2} = 1.41$$

$$\theta = \cos^{-1} \left[\frac{\vec{F} \cdot \vec{G}}{|\vec{G}| |\vec{F}|} \right] = \cos^{-1} \left[\frac{-3+4}{5 \times 1.41} \right]$$

$$= \cos^{-1} \left(\frac{1}{7.07} \right) = \cos^{-1}(0.14) = 81.9^\circ$$

$$\text{Component} = |\vec{G}| \cos \theta = 1.41 \cos(81.9) = 0.20$$

Quiz #2 Ch.#3 T133 Phys101.02-v6

Student ID:..... Student Name:.....

Q#1: Two vectors are given by: $\vec{P} = -1.5\hat{i} + 2.0\hat{j}$, $\vec{Q} = 1.0\hat{j}$. The angle that the vector $2\vec{P} - \vec{Q}$ makes with the positive x-axis is: (A: 135°)

$$\begin{aligned}\vec{A} &= 2\vec{P} - \vec{Q} = 2(-1.5\hat{i} + 2.0\hat{j}) - 1.0\hat{j} = -3.0\hat{i} + 4.0\hat{j} - 1.0\hat{j} \\ &= -3\hat{i} + 3\hat{j} \\ \theta &= \tan^{-1}\left(\frac{3}{-3}\right) = \tan^{-1}(-1) = -45^\circ = 180 - 45 = 135^\circ\end{aligned}$$

Q#2: Three vectors are given as: $A = -3.0\hat{i}$; $B = -5.0\hat{k}$ and $C = 2.0\hat{j}$. The value of $A \cdot (B \times C)$ is: (Ans -30)

$$\begin{aligned}B \times C &= -5.0\hat{k} \times 2.0\hat{j} = -10.0(\hat{k} \times \hat{j}) = 10(-\hat{i}) \\ &= +10\hat{i} \\ A \cdot (B \times C) &= (-3.0\hat{i}) \cdot (+10\hat{i}) = -30\end{aligned}$$

Quiz #2 Ch.#3 T133 Phys101.02-v7

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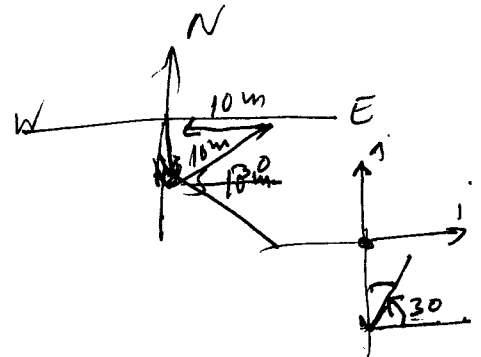
Q#1: Starting from the origin, a boy walks 3.5 m south, then 10 m at 30 degrees North of East, and finally 10 m West. Find the resultant displacement vector. Take East along the positive x-axis and North along the positive y-axis. (Ans: $(-1.34 i - 1.5 j)$ m.)

$$\vec{A} = -3.5 \vec{j}; \quad \vec{B} = 10 \cos 30 \vec{i} + 10 \sin 30 \vec{j}; \quad \vec{C} = -10 \vec{i}$$

$$D = \vec{A} + \vec{B} + \vec{C}$$

$$= -3.5 \vec{j} + 8.66 \vec{i} + 5 \vec{j} - 10 \vec{i}$$

$$= -1.34 \vec{i} + 1.5 \vec{j}$$



Q2.. Let $A=2.0i+ 3.0 k$ and $B=2.0 i+ k$. The vector $D=(A-B) \times A$ is: (Ans: $-8.0 j$)

$$\vec{A} - \vec{B} = 2\vec{i} + 3\vec{k} - (2\vec{i} + \vec{k}) = 2\vec{k}$$

$$D = (\vec{A} - \vec{B}) \times \vec{A} = 2\vec{k} \times (2\vec{i} + 3\vec{k})$$

$$= 4(\vec{k} \times \vec{i}) + 6(\underbrace{\vec{k} \times \vec{k}}_{=0}) = 4\vec{j}$$

Quiz #2 Ch.#3 T133 Phys101.02-v8

Student ID:..... Student Name:.....

Q1) If vector $A = 6\mathbf{i} - 7\mathbf{j}$ and vector $B = -12\mathbf{i} + 10\mathbf{j}$, what angle does vector $C = 2A - B$ make with +x-axis measured counterclockwise. (Ans: 315 deg.)

$$\begin{aligned}
 C &= 2A - B = 2(6\vec{i} - 7\vec{j}) - (-12\vec{i} + 10\vec{j}) \\
 &= 12\vec{i} - 14\vec{j} + 12\vec{i} - 10\vec{j} = 24\vec{i} - 24\vec{j} \\
 \theta &= \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-24}{+24}\right) = -45^\circ = \cancel{180} + 45 = 315^\circ
 \end{aligned}$$

Q2

Two vectors \vec{A} and \vec{B} have magnitudes 3.0 and 4.0 respectively. Their vector product is $\vec{A} \times \vec{B} = -5.0\vec{k} + 2.0\vec{i}$. Find the angle between \vec{A} and \vec{B} .

A) 27°

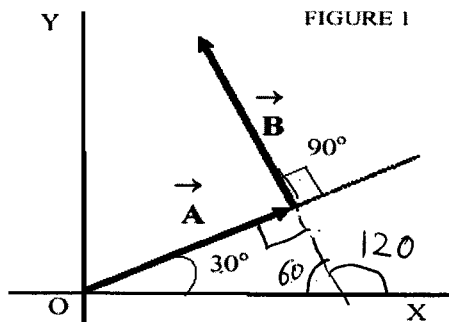
$$\begin{aligned}
 \theta &= \sin^{-1}\left(\frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}\right) = \sin^{-1}\left(\frac{\sqrt{25+4}}{3 \times 4}\right) \\
 &= \sin^{-1}\left(\frac{5.385}{12}\right) = \underline{26.6^\circ}
 \end{aligned}$$

Quiz #2 Ch.#3 T133 Phys101.02-v9

Student ID:..... Student Name:.....

Q1: The two vectors A and B shown in Fig. 1 have equal magnitudes of 10.0 m. Find the magnitude of the resultant, R, of these vectors and the angle theta it makes with the positive x-axis. (Ans: R = 14.1 m, THETA = 75 degrees)

$$\begin{aligned}
 R &= \vec{A} + \vec{B} \\
 R &= (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} \\
 &= (10\cos 30 + 10\cos 120)\vec{i} \\
 &\quad + (10\sin 30 + 10\sin 120)\vec{j} \\
 &= (8.66 + (-5))\vec{i} + (5 + 8.66)\vec{j} \\
 &= +3.66\vec{i} + 13.66\vec{j} \Rightarrow |R| = \sqrt{R_x^2 + R_y^2} = 14.1 \\
 \theta &= \tan^{-1}\left(\frac{13.66}{3.66}\right) = 75^\circ
 \end{aligned}$$



Q2. Three vectors are $A = 1.00\vec{i} + 2.00\vec{j} - 3.00\vec{k}$, $B = 3.00\vec{k}$ and $C = 6.00\vec{i} - 7.00\vec{j}$. Find $2C \cdot (A \times B)$. (Ans: 114)

$$\begin{aligned}
 \vec{A} \times \vec{B} &= (1\vec{i} + 2\vec{j} - 3\vec{k}) \times 3\vec{k} = 3(\vec{i} \times \vec{k}) + 6(\vec{j} \times \vec{k}) \\
 &= -3\vec{j} + 6\vec{i} = 6\vec{i} - 3\vec{j} \\
 2C \cdot (A \times B) &= 2(6\vec{i} - 7\vec{j}) \cdot (6\vec{i} - 3\vec{j}) \\
 &= 2(36 + 21) = 2 \times 57 = 114
 \end{aligned}$$

Quiz #2 Ch.#3 T133 Phys101.02-v10

Student ID:..... Student Name:.....

Q1: :Given the vectors $A = 3j + 6k$, $B = 15i + 21k$. Find the magnitude of vector C that satisfies equation $2A + 3C - B = 0$.
(Ans: 6.16)

$$2\vec{A} + 3\vec{C} - \vec{B} = 0 \Rightarrow \vec{C} = \frac{\vec{B} - 2\vec{A}}{3}$$

$$\vec{C} = \frac{15\vec{i} + 21\vec{k} - 2(3\vec{j} + 6\vec{k})}{3} = \frac{15\vec{i} - 6\vec{j} + 9\vec{k}}{3}$$

$$\vec{C} = 5\vec{i} - 2\vec{j} + 3\vec{k}$$

$$|\vec{C}| = \sqrt{5^2 + (-2)^2 + (3)^2} = 6.16$$

Q2

For the following three vectors, find $\vec{C} \cdot (2\vec{A} \times \vec{B})$

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j}, \quad \vec{B} = -3.00\hat{i} + 4.00\hat{j} \quad \text{and} \quad \vec{C} = 7.00\hat{i} + 3.00\hat{k}$$

A) 102

$$2(\vec{A} \times \vec{B}) = 2 \left[(2\vec{i} + 3\vec{j}) \times (-3\vec{i} + 4\vec{j}) \right]$$

$$= 2 \left[8(\vec{i} \times \vec{j}) - 9(\vec{j} \times \vec{i}) \right] = 2 \left[8\vec{k} + 9\vec{k} \right]$$

$$= 34\vec{k}$$

$$\vec{C} \cdot (2\vec{A} \times \vec{B}) = (7\vec{i} + 3\vec{k}) \cdot 34\vec{k} = 3 \times 34 = 102$$