Quiz #1 Ch.#16 T131-Sec. 7-9-v1

Student ID:......... ...... Student Name:........... .... Section # ....

Q1: A string has a mass of 0.20 g and a length of 1.6 m. A sinusoidal wave is travelling on this string, and is given by: $y(x,t) = 0.030 \sin(0.30x - 80t + 3\pi/2)$ (SI units). What is the magnitude of the tension in the string?

3/3

$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = v^2\mu$; $v = \frac{\omega}{k} = \frac{80}{0.3} = 266.7$ m/s

$\mu = \frac{m}{\ell} = \frac{0.2\times10^{-3}}{1.6} = 1.25\times10^{-4}$ kg/m

$\tau = 1.25\times10^{-4}\times(266.7)^2 = 8.89$ N

Q#2: A transverse sinusoidal wave on a string is described by the wave function $y(x,t) = 0.15 \sin(0.80x - 50t)$, where x and y are in meters and t is in seconds. The mass per unit length of the string is 12 g/m. What is the average power transmitted by the wave?
A) 21 W

4/4

$P = \frac{1}{2}\mu v \omega^2 y_m^2$, $\mu = 12$ g/m $= 12\times10^{-3}$ kg/m

$v = \frac{\omega}{k} = \frac{50}{0.8} = 62.5$ m/s

$y_m = 0.15$ m

$P = \frac{1}{2}\mu v\omega^2 y_m^2 = \frac{1}{2}\times12\times10^{-3}\times62.5\times(50)^2\times(0.15)^2 = 21.1$ W

Q#3: A string of length 2.5 m is fixed at both ends. A standing wave of frequency 100 Hz is set up on the string. The distance between two adjacent nodes is 0.50 m. What is the fundamental frequency of the string? A) 20 Hz

$f_n = 100$ Hz ; $L = 2.5$ m , $\lambda_n = 0.5\times2 = 1.0$ m

$\lambda_n = \frac{2L}{n}$

$n = \frac{2L}{\lambda_n} = \frac{2\times2.5}{1} = 5$

$f_5 = 100$

$f_1 = \frac{f_n}{n} = \frac{100}{5} = 20$ Hz

3/3

Quiz #1 Ch.#16 T131-Sec. 7-9-v2

Student ID:......... ...... Student Name:............ .... Section # ....

Q#1: An 8.00 m long wire with a mass of 10.0 g is under a tension of 25.0 N. A transverse wave for which the wavelength is 0.100 m, and the amplitude is 3.70 mm is propagating on the wire. The magnitude of the maximum transverse acceleration of a point on the wire is: A) 29.2×10^4 m/s

3/3

$|a_{y_{max}}| = \omega^2 y_m$; $\omega = vk$; $v = \sqrt{\frac{\tau}{\mu}}$; $\mu = \frac{m}{l} = \frac{0.01}{8} = 1.25\times10^{-3}$ kg/m

$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{25}{1.25\times10^{-3}}} = 141.42$ m/s ; $\omega = 141.42 \times k = 141.42\times\frac{2\pi}{\lambda}$

$\omega = 141.42\times\frac{2\pi}{0.1}$

$\omega = 141.42\times\frac{2\pi}{0.1} = 8885.8$ rad/s

$a_{y_{max}} = (8885.8)^2\times3.7\times10^{-3}$

$= 29.2\times10^4$ m/s^2

Q#2 Two identical waves having a phase difference of 0.127 λ and moving in the same direction along a stretched string. They interfere with each other and the amplitude of the resultant wave is 14.0 mm. What is the amplitude of each wave? A) 7.60 mm

(3/3)

$y_m' = 2y_m\cos(\phi/2)$, $y_m' = 14$ mm.

$\phi = 0.127\lambda = 0.127\times2\pi = 0.798$ rad $= 45.7^0$

$y_m = \frac{y_m'}{2\cos(\phi/2)} = \frac{14}{2\cos(\frac{45.7}{2})} = 7.60$ mm

Q#3: A string with a linear mass density of 0.0350 kg/m and a mass of 0.0140 kg is clamped at both ends. Under what tension in the string will it have a standing wave with a fundamental frequency of 110 Hz? A) 271 N

(4/4)

$f_1 = 110$ Hz , $f_1 = \frac{v}{2L} \Rightarrow v = f_1\times2L = \sqrt{\frac{\tau}{\mu}}$

$(f_1\times2L)^2 = \frac{\tau}{\mu}$ | $\mu = \frac{m}{L}$, $L = \frac{m}{\mu}$

$\tau = \mu\times f_1^2\times4L^2$

$= \mu\times f_1^2\times4\times\left(\frac{m}{\mu}\right)^2$

$\tau = 0.035\times(110)^2\times4\times\left(\frac{0.014}{0.035}\right)^2 = 271.04$ N

Quiz #1 Ch.#16 T131-Sec. 7-9-v3

Student ID:.............. Student Name:...................... Section # ....

Q#1: A uniform wire, having a mass of 0.4 kg and length of 6.5 m, is connected to a pulse generator. The tension is maintained in the wire by suspending a 3.5 kg mass on the other end. Find the time it takes a pulse to travel from a pulse generator to the other end.(0.28 s)

$t = \frac{L}{v} = \frac{L}{\sqrt{\frac{\tau}{\mu}}}$; $\tau = mg = 3.5 \times 9.8 = 34.3\ N$

$\mu = \frac{m}{L} = \frac{0.4}{6.5} = 0.0615\ kg/m$

4/4

$t = \frac{6.5}{\sqrt{34.3/0.0615}} = 0.2752\ sec$

Q#2: Energy is transmitted at the rate of $P_0 = 10$ W by a wave of frequency f_0 on a string under tension τ_0. What is the new energy transmission rate P if the tension is increased to $4\tau_0$ and frequency is decreased to $f_0/2$? A) P= 5 W

$P = \frac{1}{2}\mu v \omega^2 y_m^2 = \frac{1}{2}\mu\sqrt{\frac{\tau}{\mu}} \times (2\pi f)^2 \times y_m^2$

3/3

$P = \frac{1}{2}\sqrt{\mu\tau} \times 4\pi^2 f^2 \times y_m^2$

$\frac{P_1}{P_0} = \frac{\frac{1}{2}\sqrt{\mu\tau_1} \times 4\pi^2 \times f_1^2 \times y_m^2}{\frac{1}{2}\sqrt{\mu\tau_0} \times 4\pi^2 \times f_0^2 \times y_m^2} = \sqrt{\frac{\tau_1}{\tau_0}} \times \left(\frac{f_1}{f_0}\right)^2 = \sqrt{\frac{4\tau_0}{\tau_0}} \times \left(\frac{f_0/2}{f_0}\right)^2$

$= 2 \times \frac{1}{4} = \frac{1}{2} \Rightarrow P_1 = \frac{1}{2}P_0 = 5W$

Q#3: A standing wave is established on a 3.0 m long string fixed at both ends. The string vibrates in three loops with an amplitude of 1.0 cm. If the wave speed is 100 m/s, what is the frequency? (A) 50 Hz

$L = 3.0\ m$

$L = n\frac{\lambda}{2} = 3 \times \frac{\lambda}{2}$

3/3

$\lambda = \frac{2L}{3} = \frac{2 \times 3}{3} = 2.0\ m$

$f = \frac{v}{\lambda} = \frac{100}{2} = 50\ Hz$

Quiz #1 Ch.#16 T131-Sec. 7-9-v4

Student ID:......... ...... Student Name:......... .... Section # ....

Q#1: A stretched string of mass 2.0 g and length 10 cm, carries a wave having the following displacement wave: $y(x,t) = 0.05\sin(2\pi x - 400\pi t)$, where x and y are in meters and t is in seconds. What is the tension in the string? (800 N)

4/4

$v = \sqrt{\frac{\tau}{\mu}} = \frac{\omega}{k}$, $\mu = \frac{m}{L} = \frac{2\times10^{-3}}{0.1} = 0.02$ kg/m

$\tau = \mu \cdot \left(\frac{\omega}{k}\right)^2 = 0.02 \times \left(\frac{400\pi}{2\pi}\right)^2 = 800$ N

Q#2: Two identical traveling waves, with a phase difference φ, are moving in the same direction. If they are interfering and the combined wave has an amplitude 0.5 times that of the common amplitude of the two waves, calculate φ (in radians).(2.64 radians)

3/3

$y_m' = 2y_m \cos(\phi/2)$, $y_m'/y_m = 0.5$

$\phi = 2\cos^{-1}\left(\frac{y_m'}{2y_m}\right) = 2\cos^{-1}\left(\frac{1}{2}\cdot\frac{y_m'}{y_m}\right)$

$= 2\cos^{-1}(0.25) = 2\times75.52$ deg $= \frac{2\times75.52}{180}\times\pi$

$= 2.64$ rad

Q3 A vibrator having a frequency of 200 Hz generates a standing wave of six loops with amplitude of in a string clamped at both side. If the speed of the wave on the string is 100 m/s, what is the length of the string (1.5 m)

3/3

$f_n = 200$ HZ

For six loops $n = 6$ then $L = n\frac{\lambda}{2} = 3\lambda$

$\lambda = v/f$, $v = 100$ m/s

$L = 3\lambda = 3\left(\frac{v}{f}\right) = \frac{3\times100}{200} = 1.5$ m

Quiz #1 Ch.#16 T131-Sec. 7-9-v5

Student ID:............ Student Name:........... .... Section # ....

Q1. A stretched string has a length of 2.00 m and mass of 1.56 g. A transverse sinusoidal wave is travelling on this string, and is given by: $y(x, t) = 0.100 \sin(3.00x - 144t)$ where x and y are in meters and t is in seconds. What is the magnitude of the tension in the string? (1.80 N)

$v = \sqrt{\frac{\tau}{\mu}} = \omega/k$; $\mu = \frac{1.56\times10^{-3}}{2} = 0.78\times10^{-3}$ kg/m

$\tau = \mu\left(\frac{\omega^2}{k^2}\right)$

$\tau = 0.78\times10^{-3} \times \left(\frac{144}{3}\right)^2 = 1797.1\times10^{-3} = 1.797\ N$

3/3

Q#2: A sinusoidal string wave has an amplitude of 6.0 mm and a frequency of 20 Hz. If the string has a linear mass density of 100 g/m and is under a tension of 10 N, what is the average power transmitted by the wave? A) 0.28 W.

$P = \frac{1}{2}\mu v \omega^2 y_m^2$, $\mu = 100\ g/m = 0.1\ kg/m$, $y_m = 6mm = 6\times10^{-3}$

$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{10}{0.1}} = \sqrt{100} = 10\ m/s$, $\omega = 2\pi f = 2\pi\times20 = 125.7$ Hz

$P = \frac{1}{2}\mu v \omega^2 y_m^2 = \frac{1}{2}\times0.1\times10\times(125.7)^2\times(6\times10^{-3})^2 = 0.284\ W$

4/4

Q#3 A string that is stretched between fixed supports oscillates in a third-harmonic standing wave pattern. The displacement of the wave is given by $y(x,t) = (0.10)\sin(\pi x/5)\cos(12\pi t)$, where x and y are in meters, and t is in seconds. What is the length of the string? A) 15 m

$n=3$, $\lambda_n = \frac{2L}{3}$, $L = \frac{3\lambda_n}{2} = \frac{3}{2}\frac{2\pi}{k} = \frac{3\pi}{k} =$

$= \frac{3\pi}{k} = \frac{3\pi}{\pi/5} = 15m$

3/3

Quiz #1 Ch.#16 T131-Sec. 7-9-v6

Student ID:......... ...... Student Name:............ .... Section # ....

Q#1: A sinusoidal string wave traveling in the negative x direction has an amplitude of 0.20 mm and a frequency of 10 Hz. If the string has a linear mass density 0.50 kg/m and is under a tension of 10 N, what is the equation of the wave? [y = (0.20 mm) sin(14 x + 63 t).]

4/4

$\omega = 2\pi f = 2\pi \times 10 = 20\pi = 62.83\ sec^{-1}$.

$k = \frac{\omega}{v} = \omega \times \sqrt{\frac{\mu}{\tau}} = 20 \times \pi \times \sqrt{\frac{0.5}{10}} = 14.05\ m^{-1}$

$y(x,t) = y_m \sin(kx + \omega t) = 0.20\,mm \sin(14x + 63t)$

Q#2: Two identical sinusoidal waves, each having amplitude *ym*, are traveling in the same direction on the same stretched string. What phase difference between them will give a resultant wave whose amplitude is 0.5 *ym*? (151 degrees)

3/3

$y_m' = 0.5 y_m = 2 y_m \cos(\phi/2)$, $\frac{y_m'}{y_m} = 0.5$

$\phi = 2\cos^{-1}\left(\frac{y_m'}{2 y_m}\right) = 2\cos^{-1}\left(\frac{1}{2} \times 0.5\right) = 2\cos^{-1}(0.25)$

$= 2 \times 75.52 = 151$ degrees (2.64 rad)

Q#3: A string with a length of 2.5 m, fixed at both ends, has two successive resonances at frequencies of 112 Hz and 140 Hz. Determine the wavelength of the 140 Hz resonance. A) 1.0 m

3/3

$f_1 = f_{n+1} - f_n = 140 - 112 = 28\ Hz$

$f_1 = \frac{v}{2L} \Rightarrow v = 2L f_1 = 2 \times 2.5 \times 28 = 140\ m/s$

$f' = 140\ Hz$, $\lambda' = \frac{v}{f'} = \frac{140}{140} = 1.0\ m$

Quiz #1 Ch.#16 T131-Sec. 7-9-v7

Student ID:......... ...... Student Name:........... .... Section # ....

Q#1: A wave with an amplitude of 1.0 cm and wavelength 2.5 m is generated on a string with a linear density of 20 g/m that is under a tension of 5.0 N. What is the maximum transverse speed of a point on the string? (0.40 m)

$\omega = kv = \frac{2\pi}{\lambda} v = \frac{2\pi}{\lambda}\sqrt{\frac{\tau}{\mu}}$

$u_{max} = \omega y_m$

$= \frac{2\pi}{2.5} \times \sqrt{\frac{5}{20\times10^{-3}}} \times y_m \qquad \mu = 20 g/m = 20\times10^{-3}$ kg/m

$u_{max} = 39.74 \times 0.01 = 0.397$ m/s $= 0.40$ m/s

4/4

Q#2: What phase difference (in wavelength λ) between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.75 times that of the common amplitude of the two combined waves? (0.16 λ)

$y'_m = 2 y_m \cos(\phi/2) , \quad \frac{y'_m}{y_m} = 1.75$

$\phi = 2\cos^{-1}\left(\frac{1}{2}\frac{y'_m}{y_m}\right) = 2\cos^{-1}\left(\frac{1}{2}\times1.75\right)$

$= 2\cos^{-1}(0.875) = 57.91°$

$= 1.01$ rad $= \frac{1.01}{2\pi} \times \lambda$

$\phi = 0.16\lambda$

3/3

Q#3: Vibrations with frequency 600 Hz are established on a string of length 1.33 m that is clamped at both ends. The speed of waves on the string is 400 m/s. How many antinodes are contained in the resulting standing wave pattern? A) 4

$f_n = 600$ Hz ; $f_n = \frac{nv}{2L} = \frac{n\times400}{2\times1.33} = 600$

$n = \frac{2\times1.33\times600}{400} = 3.99 = 4$

3/3

Quiz #1 Ch.#16 T131-Sec. 7-9-v8

Student ID:……… …… Student Name:………… ………. Section # ……….

Q#1: The function y(x,t) = 15.0 cos (πx- 20 πt) with x and y in meters and t in seconds, describes a wave on a taut string. What is the mass of one meter of the string if the tension in the string is 40.0 N? (100g)

3/3

$v = \frac{\omega}{k} = \sqrt{\frac{\tau}{\mu}} \Rightarrow \mu = \left(\frac{k}{\omega}\right)^2 \tau \; ; m = \mu \times l$

$m = \left(\frac{k}{\omega}\right)^2 \tau \times l = \left(\frac{\pi}{20\pi}\right)^2 \times 40 \times 1 = 0.1 \text{ kg} = 100 \text{ g}$

A transverse sinusoidal travelling wave on a stretched string is given by: *y (x,t)* = 0.00230 *sin* (6.98 *x* + 742 *t*), where *x* and *y* are in meters, and *t* is in seconds. The length of the string is 1.35 m and its mass is 3.38 g. What is the average power carried by the wave? A) 0.387 W

4/4

$P = \frac{1}{2}\mu v \omega^2 y_m^2 \Rightarrow \mu = \frac{3.38\times10^{-3}}{1.35} = 2.50\times10^{-3} \text{ kg/m}$

$v = \omega/k = \frac{742}{6.98} = 106.30 \text{ m/s}$

$P = \frac{1}{2} \times 2.5\times10^{-3} \times 106.3 \times (742)^2 \times (0.0023)^2 = 0.387 \text{ W}$

Q#3: A string that is stretched between fixed supports has resonant frequencies of 385 and 430 Hz, with no intermediate resonant frequencies. What is the frequency of the seventh harmonic? A) 315 Hz

3/3

$f_1 = f_{n+1} - f_n = 430 - 385 = 45 \text{ Hz}$

$f_7 = 7 f_1 = 7 \times 45 = 315 \text{ Hz}$

Quiz #1 Ch.#16 T131-Sec. 7-9-v9

Student ID:……… …… Student Name:………… ………. Section # ……….

Q#1: A wave on a stretched string is described by the displacement wave $y = 0.400 \sin(300t - 15.0x)$, where x and y are in meters and t is in seconds. What is the phase difference between two points on the string 21.0 cm apart? A) 3.15 rad

$\Delta x = 21.0\ cm = 0.21\ m$

$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi = \frac{\Delta x}{2\pi / k} \times 2\pi = k \times \Delta x$

$\Delta\phi = k \times \Delta x = 15 \times 0.21 = 3.15\ rad.$

Q#2: Two sinusoidal waves have the same frequency, the same amplitude y_m, and travel in the same direction in the same medium. If the amplitude of the resultant wave is 1.8 y_m, the phase difference between the two waves in wavelengths is A) 0.14 wavelengths

$\phi = 2\cos^{-1}\left(\frac{y_m'}{2y_m}\right) = 2\cos^{-1}\left(\frac{1}{2} \times \frac{1.8 y_m}{y_m}\right)$

$= 2\cos^{-1}(0.9) = 51.68^0 = \frac{51.68}{360} \times \lambda$

$= 0.143\ \lambda$

Q#3: Q5. Standing waves are produced by the interference of two traveling sinusoidal waves, each of frequency 100 Hz. The distance from the second node to the fifth node is 60 cm. The wavelength of each of the two original waves is: A) 40 cm

Distance from 2nd node to 5th node $= \frac{3\lambda}{2} = 60\ cm = 0.6\ m$

$\lambda = 40\ cm$