13. Signal Transmission

We now turn to the problem of transmitting pulse signals from one part of the electronics system to the other, or, more specifically, the interconnecting cables. This may seem somewhat trivial, at first, but this will be shown to be otherwise.

The goal of signal transmission, of course, is two-fold: (1) get the signal from point A to point B, and (2) preserve the information in the signal. Recalling that a pulse generally consists of a continuous spectrum of frequencies from 0 to infinity, this would mean that our interconnecting cable would have to be capable of transmitting an infinite range of frequencies uniformly and coherently over the required distance – in most systems, a few meters. Such an ideal cable, of course, does not exist. Stray capacitances, inductances and resistance, inherent in any configuration of conductors, will invariably attenuate some frequencies more than others, causing a distortion of the pulse at the receiving end. Indeed, sending a *fast* pulse signal through simple wire connections, for example, already results in intolerable distortion after only a few centimeters!

In practice, of course, it is not necessary to transmit an infinite range of frequencies. The Fourier spectrum of a rectangular pulse of width T is largely contained in the region $\Delta f \simeq 1/T$, and, as we have seen, most of the information will be reproduced if only this range is kept. This is not to imply that our problems are over, however. In deed, for a fast 2 or 3 ns pulse, this means uniformly transmitting all frequencies up to several hundred MHz. This is still a very large range and it is by no means obvious how transmission of even this finite range can be performed without attenuation of some frequencies. Fortunately, the theory of pulse signal transmission has been developed for some time now and has allowed the design of *transmission* lines which permit low-distortion transmission over long distances.

In nuclear electronics, the standard transmission line is the coaxial cable. These cables offer a number of advantages as opposed to other designs and our discussion will be focused primarily on this cable type, although much of what will be presented is generally applicable to other transmission lines as well. Since it is the physicist who makes these interconnections, (the design of the module circuits being left to the electronics engineer!), it is, of course, extremely important that he understand how these signals are transmitted and to recognize the problems which arise.

13.1 Coaxial Cables

The basic geometry of a coaxial transmission line is that of two concentric cylindrical conductors separated by a dielectric material. A cutaway section of a typical cable showing its construction is illustrated in Fig. 13.1. The outer cylinder, which carries the return current, is generally made in the form of wire braid, while the dielectric material is usually of polyethylene plastic or teflon, although other materials are sometimes

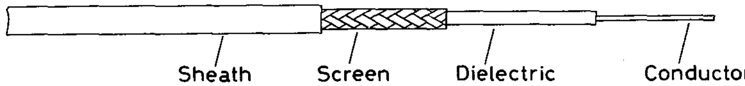


Fig. 13.1. Cut-away view of a coaxial cable

used. The entire cable is protected by a plastic outer covering. One advantage of this type of construction is that the outer cylindrical conductor, besides serving as the ground return, also shields the central wire from stray electromagnetic fields. Frequencies down to ≃ 100 kHz are effectively attenuated in most standard cables. A variety of cable sizes and designs are available commercially from several manufacturers and are briefly summarized in Table 13.1. The most commonly used cables are RG-58C/U (50 Ω) for fast signals and RG-58/U (93 Ω) for spectroscopy work. In recent years, however, the miniature RG-174/U cables have gained particular popularity in nuclear and high energy physics. Seventy-five ohm cable (RG59/U) is also used for high voltage transmission.

Table 13.1. Some common coaxial cable types and their characteristics (data from *LeCroy catalog* [13.1])

Type [RG]	Delay [ns/m]	Diameter [cm]	Capacitance [pF/m]	Max. operating voltage [kV]	Remarks
50 Ω, single braided cables:					
58U	5.14	0.307	93.5	1.9	Standard cable for fast NIM electronics
58A/U	5.14	0.305	96.8	1.9	
58C/U	5.06	0.295	93.5	1.9	
174/U	5.14	0.152	98.4	1.5	Miniature cable for fast NIM electronics
213/U	5.06	0.724	96.8	5.0	Formerly RG-8A/U
215/U	5.06	0.724	96.8	5.0	Same as 213/U but with armor; formerly RG-10A/U
218/U	5.06	1.73	96.8	11.0	Large, low attenuation cable formerly RG-17A/U
219/U	5.06	1.73	96.8	11.0	Same as 218/U but with armor; formerly RG-18A/U
220/U	5.06	2.31	96.8	14.0	Very large, low attenuation formerly RG-19A/U
221/U	5.06	2.31	96.8	14.0	Same as above but with armor; formerly RG-20A/U
50 μ, double braided cables:					
55B/U	5.06	0.295	93.5	1.9	Small size, flexible cable
221/U	5.06	0.470	93.5	3.0	Small size, microwave cable; formerly RG-5B/U
214/U	5.06	0.724	98.4	5.0	Formerly RG-9BU
217/U	5.06	0.940	96.8	7.0	Power transmission cable; formerly RG 14A/U
224/U	5.06	0.940	96.8	7.0	Same as 217/U but with armor; formerly RG-74A/U
223/U	5.06	0.295	93.5	1.0	formerly RG-55A/U
High voltage cables:					
59/U	5.14	0.381	68.9	2.3	$Z = 73\ \Omega$, standard HV cable for detectors
59B/U	5.14	0.381	67.3	2.3	$Z = 75\ \Omega$

Table 13.1 (continued)

Type [RG]	Delay [ns/m]	Diameter [cm]	Capacitance [pF/m]	Max. operating voltage [kV]	Remarks
93 Ω cables:					
62/U	4.0	0.635	44.3	0.75	Standard cable for slow NIM signals in spectroscopy work
62A/U	4.0	0.632	44.3	0.75	Standard cable for slow NIM signals
High temperature, 50 Ω single braided cables:					
178B/U	4.7	0.086	95.1	1.0	Miniature size cable
179B/U	4.7	0.160	65.6	1.2	
196A/U	4.7	0.086	–	1.0	Teflon dielectric
211A/U	4.7	1.575	95.1	7.0	Operation between −55° – +200 °C; formerly RG-117A/U
228A/U	4.7	1.575	95.1	7.0	Same as 211A/U but with armor; formerly RG-118A/U
303/U	4.7	0.295	93.5	1.9	
304/U	4.7	0.47	93.5	3.0	
316/U	4.7	0.15	–	1.2	Miniature cable
High temperature, 50 Ω double braided cables:					
115/U	4.7	0.635	96.8	5.0	Used where expansion and contraction are a problem
142B/U	4.7	0.295	93.5	1.9	Small-size flexible cable
225/U	4.7	0.724	96.8	5.0	Operation between −55° – +200 °C; formerly RG-87A/U
227/U	4.7	0.724	96.8	5.0	Same as 225/U but with armor; formerly RG-116/U

As will be seen in the following sections, pulses signals are transmitted through a coaxial line as a traveling wave. As such the coaxial line is nothing more than a wave guide.[1] In electronics, however, it is customary to view the coaxial cable as a circuit element and to consider the voltage and current in the cable rather than the E and B fields, which is perhaps more familiar to the physicist. The former approach is, of course, the more practical since voltage and current are directly accessible. We will therefore take this point of view in our discussion.

13.1.1 Line Constituents

By virtue of its geometrical configuration, (two conductors separated by a dielectric), coaxial cables necessarily contain a certain self-capacitance and inductance. From electromagnetic theory, it is easy to show that for two long concentric cylinders, these components are

[1] Signals are transmitted in degenerate TEM mode. The next mode begins at frequencies well above those of interest and does not interfere for our purposes.

$$L \simeq \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \text{ [H/m]} \quad = 0.2\, K_{\mathrm{m}} \ln\left(\frac{b}{a}\right) \text{ [µH/m]}$$

$$C \simeq \frac{2\pi\varepsilon}{\ln(b/a)} \text{ [F/m]} \quad = \frac{55.6\, K_{\mathrm{e}}}{\ln(b/a)} \text{ [pF/m]} \; , \tag{13.1}$$

where a and b are the radii of the inner and outer cylinders respectively, μ and ε the permeability and permittivity of the insulating dielectric, $K_{\mathrm{e}} = \varepsilon/\varepsilon_0$ and $K_{\mathrm{m}} = \mu/\mu_0$, the permittivity and permeability relative to the vacuum. For nonferromagnetic materials, of course, $K_{\mathrm{m}} \simeq 1$. Typical values for L and C are on the order of $\simeq 100$ pF/m and a few tenths of µH/m.

In any real cable, however, there also exists a certain resistivity due to the fact that the conductors are not perfect, and a certain conductivity across the dielectric due to its "*imperfectness*" as an insulator. These components, like the capacitance and inductance, are distributed uniformly along the cable length and are generally small compared to the capacitive component. Nevertheless, for most applications, we can approximately represent a unit length of cable by the *lumped* circuit shown in Fig. 13.2. L and C are, respectively, the series inductance and capacitance per unit length, while R is the resistance per unit length and G is the conductance per unit length of the dielectric, represented here as a parallel resistance of $1/G$ Ω. These latter two quantities, as will be seen later, are responsible for signal losses in the cable. In the ideal case of a perfect lossless cable, R and G are zero.

13.2 The General Wave Equation for a Coaxial Line

With the aid of Fig. 13.2, we can now derive an equation for the voltage, V and the current, I, in the cable. Consider, therefore, a small unit length of cable, Δz, and let us calculate the difference ΔV and ΔI across this small distance,

$$\Delta V(z,t) = -R\,\Delta z\, I(z,t) - L\,\Delta z \frac{\partial I}{\partial t}(z,t)$$

$$\Delta I(z,t) = -G\,\Delta z\, V(z,t) - C\,\Delta z \frac{\partial V}{\partial t}(z,t) \, . \tag{13.2}$$

Dividing by Δz and taking the limit $\Delta z \to 0$, we find the differential equations

$$\frac{\partial V}{\partial z} = -RI - L\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t} \, . \tag{13.3}$$

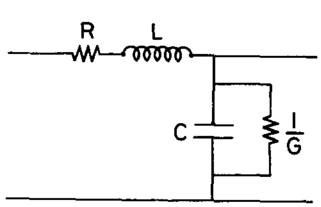


Fig. 13.2. Equivalent circuit for a unit length of transmission line

By differentiating with respect to z and t, and substituting, the equations may be uncoupled to give

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV \tag{13.4}$$

and an identical equation for I. This is the general wave equation for a coaxial cable (or, in fact, any other type of transmission line whose constituents are represented by Fig. 13.2).

13.3 The Ideal Lossless Cable

Before tackling the solutions to (13.4), let us first consider the simpler case of an ideal lossless cable where R and G are zero. For relatively short lengths of cable of a few meters or so, this, in fact, is a good approximation since the effects of R and G will be negligible for most purposes. The last two terms on the right-hand side of (13.4) vanish then leaving

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} . \tag{13.5}$$

This can be recognized as the well-known wave equation.

Suppose now a simple sinusoidal voltage in time (i.e., one Fourier component), $V = V(z)\exp(i\omega t)$ is impressed on the cable. Substitution into (13.5) then yields

$$\frac{d^2 V}{dz^2} = -\omega^2 LC\, V = -k^2 V , \tag{13.6}$$

where we have set $k^2 = \omega^2 LC$. The space solutions are then of the form

$$V(z) = V_1 \exp(-kz) + V_2 \exp(kz)$$

which results in

$$V(z,t) = V_1 \exp[i(\omega t - kz)] + V_2 \exp[i(\omega t + kz)] . \tag{13.7}$$

This represents two waves, one traveling in the $+z$ direction, and the other in the opposite direction, $-z$. This second wave corresponds to a reflection and its presence or absence depends on the boundary conditions for the cable under question. As we will see later, reflections play an important role for signal transmission since they can distort the form of the original signal.

Examination of (13.7) will also show that the quantity, k, is the wave number and that the velocity of propagation is

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{LC}} . \tag{13.8}$$

As long as the cable stays constant in in cross-section, the product LC is, in fact, independent of length and $LC = \mu\varepsilon$ where μ and ε are the permeability and permittivity of the dielectric. This is identical to the case for optical media. Thus, for a cable with free space as a dielectric, the velocity of propagation equals $1/\sqrt{\mu_0 \varepsilon_0} = c$, the speed of light in a vacuum.

The speed of signal propagation is more often expressed as its inverse, the time of propagation per unit length.

$$T = v^{-1} = \sqrt{LC} . \tag{13.9}$$

This quantity is known as the *delay* of the cable, and is typically on the order of $\simeq 5$ ns/m for standard 50 Ω cables currently found in the lab.

We might note here that the bandwidth of the ideal cable is infinite, that is, it is capable of uniformly transmitting all frequencies impressed upon it. In reality, of course, losses will limit this range as we will show later.

13.3.1 Characteristic Impedance

An important property of a transmission cable is its *characteristic impedance*. This is defined as the ratio of the voltage to the current in the cable, (including the phase relationship), i.e.,

$$Z_0 = \frac{V}{I} \tag{13.10}$$

which, with some manipulation of (13.3) and (13.7), can be shown to be

$$Z_0 = \sqrt{\frac{L}{C}} \tag{13.11}$$

for an ideal lossless cable. Examination will show that Z_0 has, in fact, the dimensions of impedance and is moreover purely resistive. It is, however, totally independent of the cable length and only dependent on the cross sectional geometry and the materials used. The characteristic impedance is, in fact, a rather special quantity in the sense that it cannot be measured with a normal resistive bridge, but behaves as a real impedance when connected to the output of a device. It is perhaps best interpreted as being the impedance offered to the propagation of the signal in the line.

For coaxial cables, an explicit calculation shows that

$$Z_0 = \sqrt{\frac{L}{C}} = 60 \sqrt{\frac{K_m}{K_e}} \ln\frac{b}{a} \quad [\Omega] \ , \tag{13.12}$$

where a and b are the inner and outer diameters of the conductors and K_m and K_e, the relative permeability and permittivity of the dielectric. At present, the standard cable used in fast nuclear electronics has a characteristic impedance of 50 Ω while cables of 93 Ω are generally used for slower spectroscopy pulses.

An interesting point to note is that all coaxial cables are necessarily limited to the range between $\simeq 50-200\ \Omega$ characteristic impedance. This is because of the dependence of Z_0 on the logarithm of b/a in (13.12). To construct a coaxial cable of 1000 Ω, for example, would require a diameter ratio of 10^{11} – clearly an impractical if not impossible task! Moreover, there is an optimum value for the ratio b/a of $\simeq 3.6$ which minimizes losses (see Sect. 13.6). If this ratio is kept and K_e is on the order of 2.3, as for polyethylene, then $Z_0 \simeq 50\ \Omega$.

13.4 Reflections

As we have seen from (13.7), the signal in a coaxial cable is, in general, the sum of the original signal and a reflected signal traveling in the opposite direction. For an arbitrary signal form f, we can write

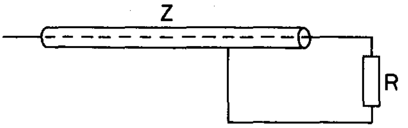


Fig. 13.3. Cable of characteristic impedance Z terminated by an impedance R

$$V = f(x - vt) + g(x + vt)\,, \tag{13.13}$$

where g is the reflected wave form. The presence of reflections can have serious consequences. Obviously, if they overlap with the original signal, interference and distortion will result. Moreover, even if no overlap occurs, it is clearly undesirable to have *echos* of the original signal bouncing back and forth in the cables leading to spurious counts and confusion. Reflections, of course, occur when a traveling wave encounters a new medium in which the speed of propagation is different. In optical media this corresponds to a change in the index of refraction. By analogy, in transmission lines, reflections occur when the characteristic impedance of the line is abruptly changed.

These reflections can be calculated by considering the boundary conditions at the interface. Consider a cable of characteristic impedance Z terminated by an impedance R, the input impedance of some device, for example. Figure 13.3 illustrates such a configuration. As a signal travels down the line, the ratio of the voltage to current must always be equal to the characteristic impedance by definition. When the interface is encountered reflections are set up which adjust the ratio of V/I to the new characteristic impedance. These reflections, however, must also be compatible with the original characteristic impedance since they will also travel back down the original line in the opposite direction. We, thus, have the conditions

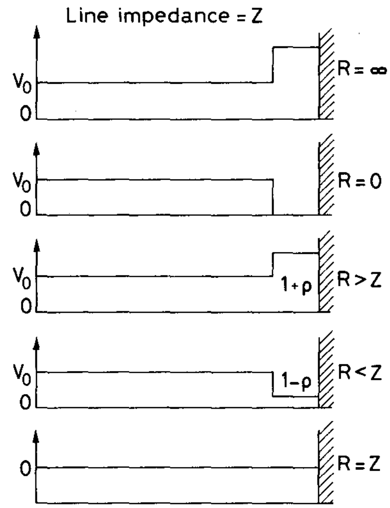
$$\begin{aligned} Z &= \frac{V_0}{I_0} \\ R &= \frac{V_0 + V_r}{I_0 + I_r} \\ Z &= \frac{V_r}{-I_r}\,, \end{aligned} \tag{13.14}$$

where V_0 and I_0 are the voltage and current of the original signal and V_r and I_r those of the reflected signal. Note the negative sign for the reflected current!

From these equations, we find

$$\rho = \frac{V_r}{V_0} = \frac{-I_r}{I_0} = \frac{R - Z}{R + Z}\,, \tag{13.15}$$

where ρ is known as the *reflection coefficient*. The polarity and amplitude of the reflected signal are thus dependent on the relative values of the two impedances. If R is greater than the cable impedance Z, then the reflection will always be of the same polarity but with an amplitude intermediate in value between 0 and the original pulse height. In the limiting case of infinite load impedance (an open circuit, for example), the reflected amplitude is equal to the incident amplitude. On the other hand, if R is smaller than the cable impedance, the reflection is opposite in polarity and intermediate in amplitude between 0 and the original. In the limit of zero load impedance (a short circuit), the reflection is equal and opposite to the incident pulse. In the special case of



◄ **Fig. 13.4.** Reflections along a transmission line. Note how the voltage level is changed due to interference from the reflection

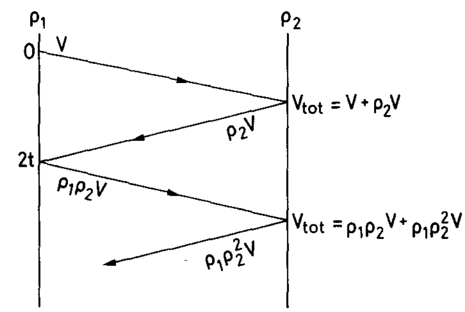


Fig. 13.5. Lattice diagram for multiple reflections

$R = Z$, ρ vanishes and there is no reflection whatsoever. Thus, only when the load and cable impedances are matched are interfering reflections avoided. The various cases are summarized in Fig. 13.4 for a simple step function pulse.

In the above example, it is easy to see what will also happen if the input end of the cable is also terminated by an impedance different from Z. Clearly, when the first reflected signal reaches the input end it will again be reflected but with some new coefficient ρ', and this again when it reaches R, etc. Multiple reflections are thus set up in the cable. An easy method of calculating the value of the signal at any time in the cable is with the simple *lattice* diagram shown in Fig. 13.5.

13.5 Cable Termination. Impedance Matching

As we have seen, signal distortion from reflections can only be avoided by matching the device impedances to the cable impedance. To a large extent, this problem has been removed by the fast NIM standard which requires that all input and output device impedances and cables be 50 Ω. However, occasions very often arise where an impedance mismatch cannot be avoided. One very common instance is when a fast signal must be viewed on an oscilloscope. Since oscilloscopes are high impedance devices ($\simeq$ 1 MΩ), direct entry of a fast NIM signal will result in an impedance mismatch and a false signal reading. (For a slow NIM signal, direct entry is compatible as given by the NIM standard, provided the cables are not too long.) In such cases, the cable can be *terminated* with an additional impedance of the appropriate value so as to adjust the total load impedance seen by the cable. For high impedance devices such as our oscilloscope, this is done by placing a resistance of 50 Ω in parallel with the device. The signal seen by the oscilloscope is then reflection free and appears as it would to a standard fast NIM module. Since the need to terminate cables arises fairly often, special 50 Ω terminators made so as to easily fit onto the cables are manufactured commercially. Figure 13.6 illustrates this method.

More generally, termination can be done in two ways: either by adding an impedance in series with the load or in parallel (shunt termination). As well, this can be