All ordinary matter consists of atoms, and each atom is made up of electrons and a nucleus. Practically all of the mass of an atom is contained in the nucleus, which consists of protons and neutrons. Because different elements contain different numbers of protons and neutrons, the atomic masses of the various elements differ. The mass of a nucleus is measured relative to the mass of an atom of the carbon-12 isotope (this isotope of carbon has six protons and six neutrons).

The mass of ^{12}C is defined to be exactly 12 atomic mass units (u), where $1\ u = 1.660\ 540\ 2 \times 10^{-27}$ kg. In these units, the proton and neutron have masses of about 1 u. More precisely,

$$\text{mass of proton} = 1.0073\ \text{u}$$

$$\text{mass of neutron} = 1.0087\ \text{u}$$

One **mole** (mol) of a substance is that amount of it that consists of Avogadro's number, N_A, of molecules. Avogadro's number is defined so that one mole of carbon-12 atoms has a mass of exactly 12 g. Its value has been found to be $N_A = 6.02 \times 10^{23}$ molecules/mol. For example, one mole of aluminum has a mass of 27 g, and one mole of lead has a mass of 207 g. But one mole of aluminum contains the same number of atoms as one mole of lead, since there are 6.02×10^{23} atoms in one mole of *any* element. The mass per atom for a given element is then given by

The mass of an atom

$$m_{\text{atom}} = \frac{\text{atomic mass of the element}}{N_A} \tag{1.2}$$

For example, the mass of an aluminum atom is

$$m_{\text{Al}} = \frac{27\ \text{g/mol}}{6.02 \times 10^{23}\ \text{atoms/mol}} = 4.5 \times 10^{-23}\ \text{g/atom}$$

Note that 1 u is equal to N_A^{-1} g.

EXAMPLE 1.2 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.7 g/cm³) has a volume of 0.20 cm³. How many aluminum atoms are contained in the cube?

Solution Since density equals mass per unit volume, the mass of the cube is

$$m = \rho V = (2.7\ \text{g/cm}^3)(0.20\ \text{cm}^3) = 0.54\ \text{g}$$

To find the number of atoms, N, we can set up a proportion using the fact that one mole of aluminum (27 g) contains 6.02×10^{23} atoms:

$$\frac{6.02 \times 10^{23}\ \text{atoms}}{27\ \text{g}} = \frac{N}{0.54\ \text{g}}$$

$$N = \frac{(0.54\ \text{g})(6.02 \times 10^{23}\ \text{atoms})}{27\ \text{g}} = 1.2 \times 10^{22}\ \text{atoms}$$

1.4 DIMENSIONAL ANALYSIS

The word *dimension* has a special meaning in physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or furlongs, it is a distance. We say its dimension is *length*.

The symbols we use in this book to specify length, mass, and time are L, M, and T, respectively. We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written $[v] = \text{L/T}$. As another exam-

TABLE 1.6 Dimensions of Area, Volume, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
cgs	cm^2	cm^3	cm/s	cm/s^2
British engineering	ft^2	ft^3	ft/s	ft/s^2

ple, the dimensions of area, A, are $[A] = L^2$. The dimensions of area, volume, speed, and acceleration are listed in Table 1.6, along with their units in the three common systems. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to derive or check a specific formula. Although you may have forgotten the details of the derivation, there is a useful and powerful procedure called *dimensional analysis* that can be used to assist in the derivation or to check your final expression. This procedure should always be used and should help minimize the rote memorization of equations. Dimensional analysis makes use of the fact that *dimensions can be treated as algebraic quantities.* That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether or not an expression has the correct form because the relationship can be correct only if the dimensions on each side of the equation are the same.

To illustrate this procedure, suppose you wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression.

The quantity x on the left side has the dimension of length. In order for the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2, and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The units of time cancel as shown, leaving the unit of length.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

when n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Since the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = LT^0$$

Since the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$(L/T^2)^n T^m = L$$

or

$$L^n T^{m-2n} = L$$

Since the exponents of L and T must be the same on both sides, we see that $n = 1$ and $m = 2$. Therefore, we conclude that

$$x \propto at^2$$

This result differs by a factor of 2 from the correct expression, which is $x = \frac{1}{2}at^2$. Because the factor 1/2 is dimensionless, there is no way of determining this via dimensional analysis.

CONCEPTUAL EXAMPLE 1.3

Does dimensional analysis give any information on constants of proportionality that may appear in an algebraic expression? Explain.

Reasoning Dimensional analysis gives the units of the proportionality constant but gives no information about its numerical value. For example, experiments show that doubling or tripling the radius of a spherical water balloon makes its mass get eight or twenty-seven times larger. Its mass is proportional to the cube of its radius. Because $m \propto r^3$, we can write $m = kr^3$. Dimensional analysis shows that the proportionality constant k must have units kg/m^3, but to determine its value requires experimental data or geometrical reasoning.

EXAMPLE 1.4 Analysis of an Equation

Show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent speeds, a is acceleration, and t is a time interval.

Solution For the speed terms, we have from Table 1.6

$$[v] = [v_0] = L/T$$

The same table gives us L/T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = (L/T^2)(T) = L/T$$

Therefore the expression is dimensionally correct. (If the expression were given as $v = v_0 + at^2$, it would be dimensionally *incorrect*. Try it and see!)

EXAMPLE 1.5 Analysis of a Power Law

Suppose we are told that the acceleration of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. How can we determine the powers of r and v?

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$L/T^2 = L^n(L/T)^m = L^{n+m}/T^m$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \quad \text{and} \quad m = 2$$

Therefore, $n = -1$, and we can write the acceleration

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2.

4. Suppose that two quantities A and B have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful: (a) $A + B$, (b) A/B, (c) $B - A$, (d) AB.
5. What level of accuracy is implied in an order-of-magnitude calculation?
6. Do an order-of-magnitude calculation for an everyday situation you might encounter. For example, how far do you walk or drive each day?
7. Estimate your age in seconds.
8. Estimate the masses of various objects around you in grams or in kilograms. If a scale is available, check your estimates.
9. Is it possible to use length, density, and time as three fundamental units rather than length, mass, and time? If so, what could be used as a standard of density?
10. An automobile tire is rated to last for 50 000 miles. Estimate the number of revolutions the tire will make in its lifetime.
11. Estimate the total length of all McDonald's french fries (laid end to end) sold in the United States in one year. How many round trips to the Moon would this equal?

PROBLEMS

Section 1.3 Density and Atomic Mass

1. Calculate the density of a solid cube that measures 5.00 cm on each side and has a mass of 350 g.
2. The mass of the planet Saturn is 5.64×10^{26} kg and its radius is 6.00×10^{7} m. (a) Calculate its density. (b) If this planet were placed in a large enough ocean of water, would it float? Explain.
3. How many grams of copper are required to make a hollow spherical shell with an inner radius of 5.70 cm and an outer radius of 5.75 cm? The density of copper is 8.93 g/cm^3.

3A. How many grams of copper are required to make a hollow spherical shell having an inner radius r_1 (in cm) and an outer radius r_2 (in cm)? The density of copper is ρ (in g/cm^3).

4. The planet Jupiter has an average radius 10.95 times that of the average radius of the Earth and a mass 317.4 times that of the Earth. Calculate the ratio of Jupiter's mass density to the mass density of the Earth.
5. Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in atomic mass units and in grams. The atomic masses are 4, 56, and 207, respectively, for the atoms given.
6. A small cube of iron is observed under a microscope. The edge of the cube is 5.00×10^{-6} cm. Find (a) the mass of the cube and (b) the number of iron atoms in the cube. The atomic mass of iron is 56 u, and its density is 7.86 g/cm^3.
7. A structural I beam is made of steel. A view of its cross-section and its dimensions is shown in Figure P1.7. (a) What is the mass of a section 1.5 m long? (b) How many atoms are there in this section? The density of steel is 7.56×10^{3} kg/m^3.
8. A flat circular plate of copper has a radius of 0.243 m and a mass of 62.0 kg. What is the thickness of the plate?

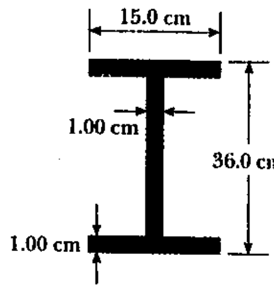

FIGURE P1.7

Section 1.4 Dimensional Analysis

9. Show that the expression $x = vt + \frac{1}{2}at^2$ is dimensionally correct, where x is a coordinate and has units of length, v is speed, a is acceleration, and t is time.
10. The displacement of a particle when moving under uniform acceleration is some function of the elapsed time and the acceleration. Suppose we write this displacement $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?
11. The square of the speed of an object undergoing a uniform acceleration a is some function of a and the displacement s, according to the expression $v^2 = ka^m s^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied only if $m = n = 1$.
12. The radius r of a circle inscribed in any triangle whose sides are a, b, and c is given by $r = [(s - a)(s - b)(s - c)/s]^{1/2}$, where s is an abbreviation for $(a + b + c)/2$. Check this formula for dimensional consistency.
13. Which of the equations below is dimensionally correct?

☐ indicates problems that have full solutions available in the Student Solutions Manual and Study Guide.

(a) $v = v_0 + ax$
(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

14. The period T of a simple pendulum is measured in time units and is

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the free-fall acceleration in units of length divided by the square of time. Show that this equation is dimensionally correct.

15. The volume of an object as a function of time is calculated by $V = At^3 + B/t$, where t is time measured in seconds and V is in cubic meters. Determine the dimension of the constants A and B.

16. The consumption of natural gas by a company satisfies the empirical equation $V = 1.5t + 0.0080t^2$, where V is the volume in millions of cubic feet and t the time in months. Express this equation in units of cubic feet and seconds. Put the proper units on the coefficients. Assume a month is 30 days.

17. Newton's law of universal gravitation is

$$F = G\frac{Mm}{r^2}$$

Here F is the force of gravity, M and m are masses, and r is a length. Force has the SI units $\text{kg}\cdot\text{m/s}^2$. What are the SI units of the constant G?

Section 1.5 Conversion of Units

18. Convert the volume 8.50 in.^{3} to m^3, recalling that 1 in. = 2.54 cm and 1 cm = 10^{-2} m.

19. A rectangular building lot is 100.0 ft by 150.0 ft. Determine the area of this lot in m^2.

20. A classroom measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.29 kg/m^3. What are (a) the volume of the room in cubic feet, and (b) the weight of air in the room in pounds?

21. A creature moves at a speed of 5.0 furlongs per fortnight (not a very common unit of speed). Given that 1.0 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in m/s. (The creature is probably a snail.)

22. A section of land has an area of 1 square mile and contains 640 acres. Determine the number of square meters there are in 1 acre.

23. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

24. A quart container of ice cream is to be made in the form of a cube. What should be the length of a side in cm? (Use the conversion 1 gallon = 3.786 liters.)

25. An astronomical unit (AU) is defined as the average distance between the Earth and Sun. (a) How many astronomical units are there in one lightyear? (b) Determine the distance from Earth to the Andromeda galaxy in astronomical units.

26. The mass of the Sun is about 1.99×10^{30} kg, and the mass of a hydrogen atom, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are there in the Sun?

27. At the time of this book's printing, the U.S. national debt is about \$4 trillion. (a) If payments were made at the rate of \$1000/sec, how many years would it take to pay off the debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If 4 trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the Earth? Take the radius of the Earth at the equator to be 6378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

28. A room measures 4.0 m × 4.0 m, and its ceiling is 2.5 m high. Is it possible to completely wallpaper the walls of this room with the pages of this book? Explain.

29. (a) Find a conversion factor to convert from mi/h to km/h. (b) Until recently, federal law mandated that highway speeds would be 55 mi/h. Use the conversion factor of part (a) to find this speed in km/h. (c) The maximum highway speed has been raised to 65 mi/h in some places. In km/h, how much increase is this over the 55 mi/h limit?

30. (a) How many seconds are there in a year? (b) If one micrometeorite (a sphere with a diameter of 1.00×10^{-6} m) strikes each square meter of the Moon each second, how many years would it take to cover the Moon to a depth of 1.00 m? (*Hint:* Consider a cubic box on the Moon 1.00 m on a side, and find how long it will take to fill the box.)

31. One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?

32. A pyramid has a height of 481 ft and its base covers an area of 13.0 acres (1 acre = 43 560 ft^2). If the volume of a pyramid is given by the expression $V = (1/3)Bh$, where B is the area of the base and h is the height, find the volume of this pyramid in cubic meters.

33. The pyramid described in Problem 32 contains approximately two million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

34. Assuming that 70 percent of the Earth's surface is covered with water at an average depth of 1 mile, estimate the mass of the water on Earth in kilograms.

35. The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 lightyears. The distance to Andromeda, the galaxy nearest our own Milky Way, is about 2.0 million lightyears. If we represent the Milky Way by a dinner plate 25 cm in diameter, determine the distance to the next dinner plate.