Math 260 – Quiz # 8b

Name: Solution Sec.#:_______ Sr #: ____________

Solve the IVP : $X' = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix} X$, $X(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 8 \\ -1 & -2-\lambda \end{vmatrix} = (2-\lambda)(-2-\lambda)+8 = \lambda^2+4 = 0 \Rightarrow \lambda = \pm 2i$ eigen values

$\underline{\lambda = 2i}$

$(A-2iI)K = 0 \Rightarrow \begin{bmatrix} 2-2i & 8 \\ -1 & -2-2i \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|c} 1 & 2+2i & 0 \\ 2-2i & 8 & 0 \end{array}\right]$

$\xrightarrow{-(2-2i)R_1+R_2} \left[\begin{array}{cc|c} 1 & 2+2i & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\Rightarrow k_1 = -(2+2i)k_2$.

Take $k_2 = -1 \Rightarrow k_1 = 2+2i$

$\Rightarrow K = \begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$. So, $Re(K) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $Im(K) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\lambda = 2i \Rightarrow \alpha = 0, \beta = 2$

$$X_1 = \left(Re(K)\cos\beta t - Im(K)\sin\beta t\right)e^{\alpha t}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}\cos 2t - \begin{bmatrix} 2 \\ 0 \end{bmatrix}\sin 2t$$

$$X_2 = \left(Im(K)\cos\beta t + Re(K)\sin\beta t\right)e^{\alpha t}$$

$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix}\cos 2t + \begin{bmatrix} 2 \\ -1 \end{bmatrix}\sin 2t$$

The general solution of the system is

$$X = c_1\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\cos 2t - \begin{bmatrix} 2 \\ 0 \end{bmatrix}\sin 2t\right) + c_2\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\cos 2t + \begin{bmatrix} 2 \\ -1 \end{bmatrix}\sin 2t\right).$$