Math 260 – Quiz # 6

Name: _____ Sec.#: _____ Sr #: _____

First we solve the associated hom. equation $y'' + y = 0$.

$$\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i \qquad [\alpha = 0, \beta = 1]$$

$y_H = C_1 \cos x + C_2 \sin x$. so $y_1 = \cos x$, $y_2 = \sin x$

We are seeking a particular solution for the given DE; $y_p = u_1 \cos x + u_2 \sin x$.

$$W = W(\cos x, \sin x) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix} = -\sin x \sec x = -\tan x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix} = \cos x \sec x = 1$$

$$u_1' = \frac{W_1}{W} = -\tan x$$

$$u_2' = \frac{W_2}{W} = 1$$

$$u_1 = \int -\tan x \, dx = \ln|\cos x|$$

$$u_2 = x$$

$$\therefore y_p = u_1 \cos x + u_2 \sin x$$

$$= \cos x \ln|\cos x| + x \sin x$$

$$y = y_H + y_p = C_1 \cos x + C_2 \sin x + \cos x \ln|\cos x| + x \sin x .$$