Math 202 Quiz # 5b

Name: Solution **Sr. #** ________ **Section #** ____

1. Find a linear differential operator that annihilates the function: $f(x) = (8 - e^x)^2$.

$$f(x) = \underset{\searrow D}{64} - \underset{\searrow D-1}{16e^{x}} + \underset{\searrow D-2}{e^{2x}}$$

$$\text{Ann}(f(x)) = D(D-1)(D-2)$$

2. Solve the following DE by undetermined coefficients: $y'' + 4y = 5\cos x + 6\sin x + 10$ —— (1)

First solve the ass. hom. DE: $y'' + 4y = 0$

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$\boxed{y_H = c_1 \cos 2x + c_2 \sin 2x}$ —— (2)

Write the DE (1) as $(D^2+4)y = 5\cos x + 6\sin x + 10$

$\text{Ann}(\text{R.H.S.}) = D(D^2+1)$

$\therefore D(D^2+1)(D^2+4)y = D(D^2+1)(5\cos x + 6\sin x + 10) = 0$

$D(D^2+1)(D^2+4)y = 0$

Solving this DE $\Rightarrow \lambda(\lambda^2+1)(\lambda^2+4) = 0 \Rightarrow \lambda = 0, \pm i, \pm 2i$

H's solution is $y = c_1 + c_2\cos x + c_3 \sin x + \underbrace{c_4 \cos 2x + c_5 \sin 2x}_{y_H}$ --(2)

Comparing (1) & (2), we get

$\boxed{y_p = A + B\cos x + C\sin x} \Rightarrow y_p' = -B\sin x + C\cos x \Rightarrow y_p'' = -B\cos x - C\sin x$

Substitute in the given DE: $y_p'' + 4y_p = 5\cos x + 6\sin x + 10$

$-B\cos x - C\sin x + 4A + 4B\cos x + 4C\sin x = 5\cos x + 6\sin x + 10$

$\Rightarrow 3B\cos x + 3C\sin x + 4A = 5\cos x + 6\sin x + 10$

Equating Coeffs. $\Rightarrow 4A = 10 \Rightarrow \boxed{A = \frac{5}{2}}$, $3B = 5 \Rightarrow \boxed{B = \frac{5}{3}}$, $3C = 6 \Rightarrow \boxed{C = 2}$

$\therefore y_p = \frac{5}{2} + \frac{5}{3}\cos x + 2\sin x$

The solution is $y = y_H + y_p$

$= c_1 \cos 2x + c_2 \sin 2x + \frac{5}{3}\cos x + 2\sin x + \frac{5}{2}$