Math 202 Quiz # 8b

Name: Solution Sec. # ____________ Ser. # ____

Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & -1 & 0 \\ 0 & 2-\lambda & 0 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)(2-\lambda)^2 = 0 \Rightarrow \lambda = 3, 2, 2$, the eigen values of A

To Find the eigen vectors:

For $\lambda = 3$ $(A - 3I)K = 0 \Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right] \xrightarrow[-R_2]{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \xrightarrow[R_2 + R_3]{-R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} k_2 = 0 \text{ \& } k_1 = -k_3 \\ \text{Take } k_3 = 1 \\ \Rightarrow k_1 = -1 \end{cases}$$

$\therefore K_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvalue corresponds to $\lambda = 3$.

For $\lambda = 2$ $(A - 2I)K = 0 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow k_1 = k_2$ & k_3 is any constant. So, we get 2 eigenvectors correspond to $\lambda = 2$ as: $K_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $K_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\longrightarrow$