Math 202 Quiz # 8c

Name: Solution Sec. # ____________ Ser. # ____

Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 9-\lambda & 1 & 1 \\ 1 & 9-\lambda & 1 \\ 1 & 1 & 9-\lambda \end{vmatrix} = 0$$

$$\xrightarrow{-R_3+R_1} \begin{vmatrix} 8-\lambda & 0 & \lambda-8 \\ 1 & 9-\lambda & 1 \\ 1 & 1 & 9-\lambda \end{vmatrix} \xrightarrow{C_1+C_3} \begin{vmatrix} 8-\lambda & 0 & 0 \\ 1 & 9-\lambda & 2 \\ 1 & 1 & 10-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\left[(9-\lambda)(10-\lambda)-2\right] = (8-\lambda)\left[\lambda^2 - 19\lambda + 88\right]$$
$$= (8-\lambda)(8-\lambda)(11-\lambda) = 0$$

$\Rightarrow \lambda = 11, 8, 8$, the eigen values of A.

To find the eigen vectors:

$\lambda = 11$: $(A - 11I)K = 0 \Rightarrow \left[\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow k_1 = k_2 = k_3$

$\therefore K_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector corresponds to $\lambda = 11$

$\lambda = 8$: $(A - 8I)K = 0 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow k_1 = -k_2 - k_3.$

- Take $k_2 = 1, k_3 = 0 \Rightarrow k_1 = -1 \Rightarrow \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$
- Take $k_2 = 0, k_3 = 1 \Rightarrow k_1 = -1 \Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ we have 2 eigenvectors $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ correspond to $\lambda = 8$.

$\longrightarrow$