Math 202 Quiz # 4a

Name: Solution Sr. # ____ Section # ____

Given that $y_1 = x$ is a solution of the DE: $(x^2+1)y'' - 2xy' + 2y = 0$
Use reduction of order to find a second solution.

Let $y = uy_1 = ux \Rightarrow y' = u + xu'$, $y'' = u' + xu'' + u'$
$= xu'' + 2u'$

substitute in the DE: $(x^2+1)y'' - 2xy' + 2y = 0$

$$(x^2+1)[xu''+2u'] - 2x[u+xu'] + 2ux = 0$$

$$(x^3+x)u'' + 2u' = 0$$

Put $w = u' \Rightarrow w' = u''$. Then we have a first-order DE:

$$(x^3+x)w' + 2w = 0$$

$$\frac{w'}{w} = -\frac{2}{x^3+x}$$

use partial fractions decomposition:

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\frac{w'}{w} = \frac{2x}{x^2+1} - \frac{2}{x}$$

$$\ln|w| = \ln|x^2+1| - 2\ln|x| + \ln|c_1|$$

$$\ln\left|\frac{wx^2}{x^2+1}\right| = \ln|c_1| \Rightarrow \frac{wx^2}{x^2+1} = c_1 \Rightarrow w = \frac{c_1(x^2+1)}{x^2}$$

$$u' = w = \frac{c_1(x^2+1)}{x^2}$$

$$u = c_1\int\left(\frac{x^2+1}{x^2}\right)dx = c_1\int\left(1+\frac{1}{x^2}\right)dx = c_1\left(x - \frac{1}{x}\right) + c_2$$

$y = ux = \left[c_1\left(x - \frac{1}{x}\right) + c_2\right]x = c_1(x^2-1) + c_2 x$. Take $c_1 = c_2 = 1$

$y = (x^2-1) + x$, where $y_1 = x$

so, $y_2 = x^2 - 1$