Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand what is meant by Type-I & Type-II regions

2. evaluate double integrals of $f(x, y)$ over these regions

Evaluating double integrals over non-rectangular regions

We will consider two types of regions

See figure in class

See figure in class

Type I region

A region bounded by

- the lines $x = a, \ x = b$

and

- the curves $y = g_1(x), \ \ y = g_2(x)$

    with $g_1(x) \leq g_2(x) \quad \forall x \in [a, b]$

Type II region

A region bounded by

- the lines $y = c, \ y = d$

and

- the curves $x = h_1(y), \ \ x = h_2(y)$

    with $h_1(y) \leq h_2(y) \quad \forall y \in [c, d]$

Double integrals over both regions evaluated as iterated integrals

If R is type I then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

If R is type II then

$$\iint_R f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$$

Important points to note:

- Outer limits always constant

- The idea of using a vertical line as explained in class

- Sometimes need to break into more integrals (see example 15.2.3)

See examples 15.2.1 to 15.2.6 done in class

Do Qs: 1-52

End of 15.2