Section 12.8 *Cylindrical and spherical coordinates*

Learning outcomes

After completing this section, you will inshaAllah be able to

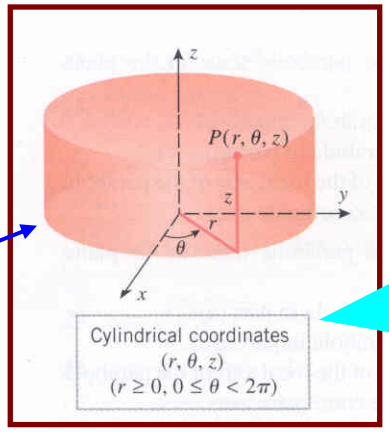
1. know cylindrical coordinate system
2. know spherical coordinate system
3. learn conversions between
    a. cylindrical and rectangular coordinates
    b. spherical and rectangular coordinates
    c. cylindrical and spherical coordinates

- In 2-dimensions, we learnt polar coordinates which gave an easier description of some curves
- Here, we introduce two coordinate systems in 3-dimensions, which give easier description of some surfaces.

What are cylindrical coordinates?

A point P in 3-space represented by coordinates (r,θ,z), where r, θ and z are as shown in the figure

See class explanation



Restriction: $r \geq 0$ and $0 \leq \theta < 2\pi$

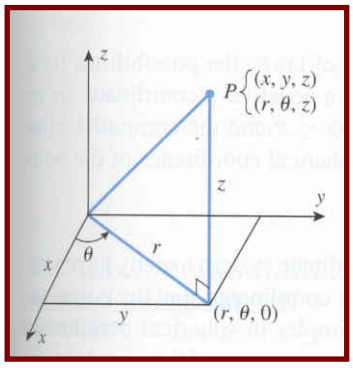
Relation between cylindrical & rectangular coordinates

- Cylindrical to rectangular: $(r,\theta,z) \rightarrow (x,y,z)$

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$

- Rectangular to cylindrical: $(x,y,z) \rightarrow (r,\theta,z)$

$$r = \sqrt{x^2 + y^2}, \quad \tan\theta = \frac{y}{x}, \quad z = z$$



Example 12.8.1 Find cylindrical coordinates of the point with rectangular coordinates $(3,-3,-7)$.

Example 12.8.2 Convert the cylindrical coordinates $\left(2, 2\pi/3, 1\right)$ into cylindrical coordinates.

Both solutions done in class

Example 12.8.3 Find the equation of ellipsoid $4x^2 + 4y^2 + z^2 = 1$ in cylindrical coordinates.

Solution Done in class

Example 12.8.4 Identify the following surfaces (which given in cylindrical coordinates)

Hint: First convert into rectangular coordinates.

(a) $r = 5$

(b) $r^2 + z^2 = 100$

(c) $z = r$

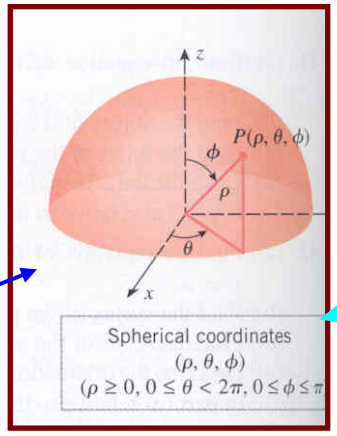
Solution Done in class

What are spherical coordinates?

A point P in 3-space represented by coordinates (ρ,θ,ϕ), where

- ρ is distance of P from origin
- θ, ϕ as shown in the figure

See class explanation



Restriction:

$\rho \geq 0$ and

$0 \leq \theta < 2\pi$

$0 \leq \phi \leq \pi$

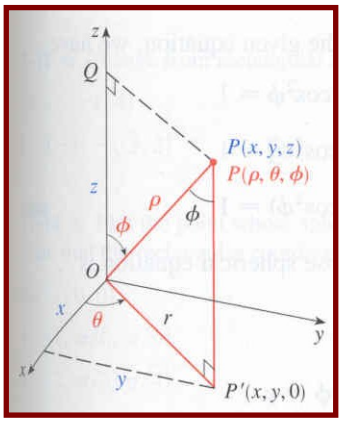
Relation between spherical & rectangular coordinates

- Spherical to rectangular: $(\rho,\theta,\phi) \to (x,y,z)$

$$x = \rho\sin\phi\cos\theta,\quad y = \rho\sin\phi\sin\theta,\quad z = \rho\cos\phi$$

- Rectangular to spherical: $(x,y,z) \to (\rho,\theta,\phi)$

$$\rho = \sqrt{x^2+y^2+z^2},\quad \tan\theta = \frac{y}{x},\quad \cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$$



Example 12.8.5

(a) Convert $\left(-1,1,-\sqrt{2}\right)$ from rectangular to spherical coordinates.

(b) Convert $\left(2,\pi/4,\pi/3\right)$ from spherical to rectangular coordinates.

Example 12.8.6 Find the equation of $x^2+y^2-z^2=1$ (of hyperboloid of one sheet) in spherical coordinates

Example 12.8.7 Identify the following surfaces

(a) $\rho = 5$ **(b)** $\rho\sin\phi = 2$

All solutions done in class

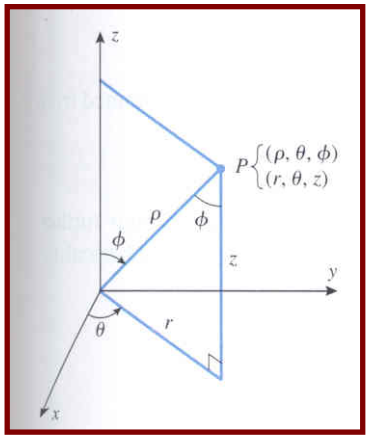
Conversion between cylindrical & spherical coordinates

- Spherical to cylindrical: $(\rho,\theta,\phi)\to(r,\theta,z)$

$$r=\rho\sin\phi,\quad \theta=\theta,\quad z=\rho\cos\phi$$

- Cylindrical to Spherical: $(r,\theta,z)\to(\rho,\theta,\phi)$

$$\rho=\sqrt{r^2+z^2},\quad \theta=\theta,\quad \tan\phi=\frac{r}{z}$$



Example 12.8.8 Convert $\left(\sqrt{6},\frac{\pi}{4},\sqrt{2}\right)$ from cylindrical to spherical coordinates.

Solution Done in class

Example 12.8.9 Convert $\left(1,\frac{7\pi}{6},\pi\right)$ from spherical to cylindrical coordinates.

Solution Done in class

Do Qs: 1-12, 15-42.

End of Section 12.8