Section 12.1 *Rectangular coordinates in 3-space; spheres; cylindrical surfaces*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand basic facts about 3-dimensional coordinates system
2. recognize and sketch spheres from the equation
3. know what is meant by a cylindrical surface
4. learn how to graph cylindrical surfaces

3-dimensional rectangular coordinate system

Generalization of 2-dimensional system

See explanation for the following in class

- **Coordinate Axes**

  Three mutually perpendicular coordinate lines X-axis, Y-axis, Z-axis (intersecting at origin).

- **Coordinate Planes**

  Three planes determined by coordinate axes XY-plane, XZ-plane, YZ-plane

- **Octants**

  Coordinate planes divide 3-space into 8 parts called octants

- **Coordinates**

  - Any point is determined through an ordered triple (a, b, c)
  - P has coordinates (a, b, c) means

    To locate P, we start from the origin, move a-units along X-axis, then b-units parallel to Y-axis and then c-units parallel to Z-axis.

Basic formulas in 3-dimensional coordinate system

Straightforward extension of similar formulas in 2-dimensions

Distance Formula

The distance between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Exercise 12.1.1 Find the distance between $P_1(2, -1, 7)$ and $P_2(1, -3, 5)$.

Answer: 3

Midpoint Formula

The coordinates of midpoint of $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Simple graphs in 3-space

- **Spheres**

Sphere with center (x_0, y_0, z_0) and radius r is given by

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$

Need
- center
- origin

Exercise 12.1.2 Find the center and radius of the sphere that has $(1,-2,4)$ and $(3,4,-12)$ as end points of a diameter. Give the equation of the sphere.

Given any equation of the form

$$x^2+y^2+z^2+Gx+Hy+Iz+J=0.$$

We can write it as

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=k$$

by completing the square

If $k>0$ then sphere

If $k=0$ then only the point (x_0, y_0, z_0)

If $k<0$ then no graph

Example 12.1.3 What is the graph of $2x^2+2y^2+2z^2+8x+12y+18=0$?

Solution Done in class.

Example 12.1.4 What region is represented by $1\le x^2+y^2+z^2\le 4$?

Solution Done in class.

Simple graphs in 3-space (contd)

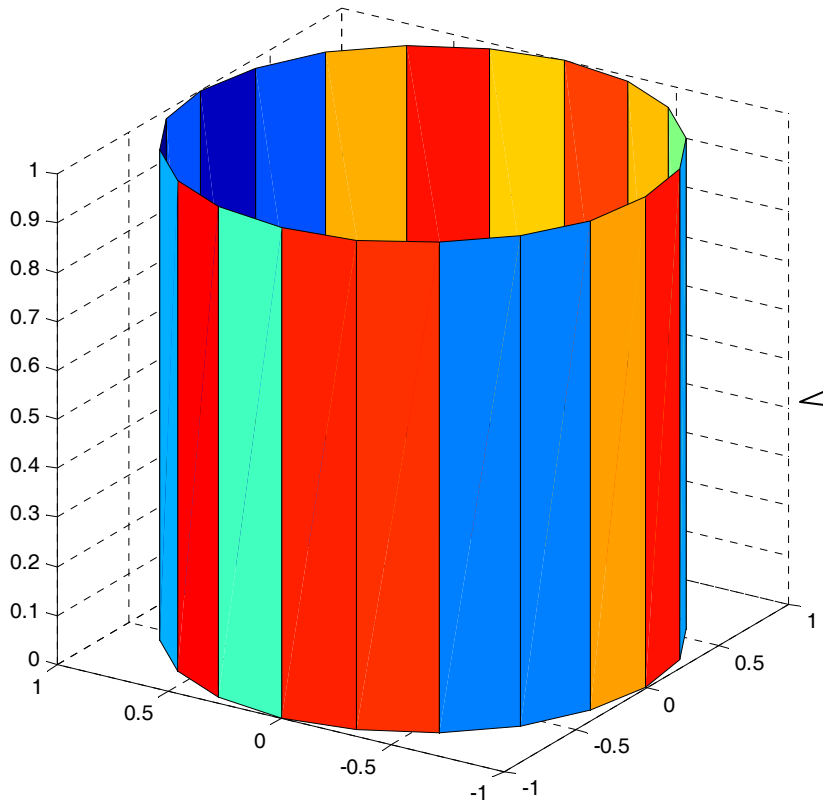
- **Cylindrical surfaces**

Let's look at an example first.

Example 12.1.5 Graph $x^2 + y^2 = 1$ in $\mathbb{R}^2$ and $\mathbb{R}^3$

Solution

- in $\mathbb{R}^2$
  - a circle of radius '1' centered at origin
- in $\mathbb{R}^3$
  - z can take any value
  - for every value of z we have a circle of radius '1' centered at Z-axis
  - we have infinitely many copies of circles sitting over each other

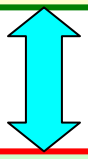


- make circle $x^2 + y^2 = 1$ in XY-plane
- translate along Z-axis

Simple graphs in 3-space (contd)

- **Cylindrical surfaces (contd)**

An equation in 2 variables represents a cylindrical surface in 3-dimensional coordinate system



To sketch

1. Graph it in 2-dimensions
2. Translate the graph parallel to the axis of missing variable

Example 12.1.6 Graph $z = x^2$ in 3-dimensional space.

Solution Done in class

End of Section 12.1

Do Qs. 5-34