Section 4.3 *How derivatives affect the shape of a graph*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. find increasing or decreasing intervals of a given function
2. use first derivative test to find relative extreme points
3. explain what is meant by concave up and concave down curves
4. explain what is a point of inflection
5. find intervals where a function is concave up or down
6. find points of inflection
7. use second derivative test to determine relative maxima or relative minima of $f(x)$ at points where $f'(x) = 0$
8. see how above concepts affect the shape of a graph

Recall from Section 4.1

- The concept of increasing or decreasing intervals of function.
- The concept of local extreme points and their relation to increasing/decreasing intervals of a function.
- The details about critical values
  - Given a function *f(x)*

The values of *x* where $\boxed{f' = 0 \text{ or } f' \text{ undefined}}$ are called critical values of *f(x)*

- These are the points where a function **may** change from increasing to decreasing or decreasing to increasing.
- These are possible relative extreme points
- Relative extrema can only occur at these points
- But we must check further

With the above ideas clear,
we next proceed to calculations and concepts
involved in this section

How to find increasing or decreasing intervals of f(x)

- Given a function $f(x)$.
- To find intervals where it is increasing or decreasing.

1) Solve $f'(x) = 0$ or undefined.

   To get points where *f(x)* may change from decreasing to increasing or increasing to decreasing.

2) Use these points to break domain into subintervals.

3) Check the sign of f' in each subinterval and make conclusion.

   - Use test points.
   - See examples for more details.

Criteria for conclusion

- $f' > 0 \quad \Rightarrow \quad f$ increasing
- $f' < 0 \quad \Rightarrow \quad f$ decreasing

See examples 1, 2 done in class

How to find local extreme points?

- Recall from Section 4.1

Relative maximum point: where f changes from increasing to decreasing

Relative minimum point: where f changes from decreasing to increasing

First derivative test for relative extrema of *f(x)*

- Find all critical values x_0 using $\boxed{f' = 0 \text{ or } f' \text{ undefined}}$
- If f' changes sign from positive to negative at x_0 then

  f has relative maximum at x_0

  f changes from increasing to decreasing

- If f' changes sign from negative to positive at x_0 then

  f has relative minimum at x_0

  f changes from decreasing to increasing

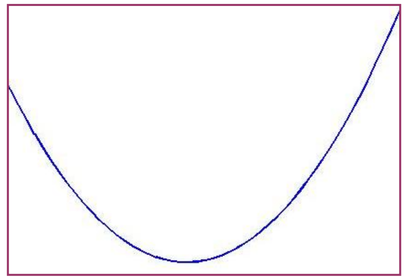
See examples 3, 4, 5, 6 done in class

Meaning of concave up, concave down

- Given a function $f(x)$.

Concave up on I

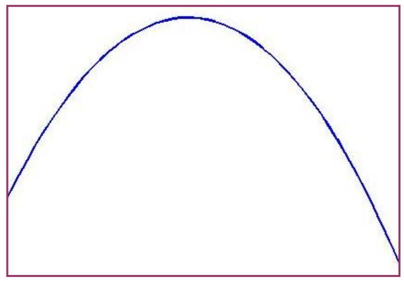
f' increasing i.e. $f''>0$ on I



tangent line below graph

Concave down on I

f' decreasing i.e. $f''<0$ on I

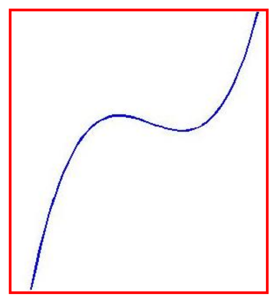


tangent line above graph

Point of inflection

- point where concavity changes
- i.e. point where f' changes from increasing to decreasing or decreasing to increasing
- i.e. point where f'' changes sign

e.g.



This can only happen at points where $f''=0$ or f'' undefined. But it is not necessary.

- So we must make table to check whether or not f'' changes sign at these points

See class explanation

See example 7, 8 done in class

How to find concavity intervals & points of inflection of f(x)

- Given a function $f(x)$.
- To find concavity intervals & points of inflection.

1) Solve $f''(x) = 0$ or undefined.

These are possible points of inflection.

2) Use these points to break domain into subintervals.

3) Check the sign of f'' in each subinterval and make conclusion.

- Use test points.
- See examples for more details.

Criteria for conclusion

- $f'' > 0 \quad \Rightarrow \quad f$ concave up
- $f'' < 0 \quad \Rightarrow \quad f$ concave down
- Points where f'' changes sign are point of inflection

See examples 9, 10, 11, 12, 13 done in class

Second derivative test

- Given a function $f(x)$.
- And we have found critical values satisfying $f'(x)=0$.
- Then we can use second derivative test to find relative extrema.

- Used to find relative extrema
- Only at critical values obtained from $f'(x)=0$

Note we can not use it for critical values obtained from $f'(x)=\text{undefined}$

Second derivative test

Suppose x_0 is a critical value obtained from $f'(x)=0$.

- If $f''(x_0)<0$ then f has a relative maximum at x_0
- If $f''(x_0)>0$ then f has a relative minimum at x_0
- If $f''(x_0)=0$ then the test is inconclusive.

See class explanation

See examples 14, 15, 16 done in class

End of 4.3