

Learning outcomes

After completing this section, you will inshaAllah be able to

1. apply the **chain rule** for differentiation
2. differentiate using **the proper form** of following differentiation formulas
 - a. the **power rule formula**
 - b. formulas for **derivatives of functions involving** a^u or e^u
 - c. formulas for **derivatives of trigonometric functions**

Chain rule for differentiation

- Recall that $(f \circ g)(x) = f(g(x))$

Chain Rule (1st form)

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

- Setting $u = g(x)$ we get the following form of the chain rule

Chain Rule (2nd form)

If y is a function of u and u is a function of x then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

See examples 1, 2, 3, 4 done in class

- Chain rule combined with all previous differentiation formulas gives their proper use.
- It is more practical to learn to directly use these formulas properly.

Next we learn proper use of all the differentiation formulas learnt until now.

The power rule formula: Proper form

Compare with power rule formula done in Section 3.1

$$\frac{d}{dx} u^n = n u^{n-1} \cdot \frac{du}{dx}$$

True for any 'n'

See examples 5, 6, 7 done in class

Derivatives of functions involving e^u

Differentiation formula for e^u

$$\frac{d(e^u)}{du} = e^u \cdot \frac{du}{dx}$$

Derivatives of functions involving a^u

Differentiation formula for a^u

$$\frac{d(a^u)}{du} = \ln a \cdot \left(a^u \frac{du}{dx} \right)$$

How?

- Using $a = e^{\ln a}$
- $\Rightarrow a^u = (e^{\ln a})^u = e^{u \ln a}$

See examples 8, 9 done in class

Differentiation formulas for trigonometric functions (proper form)

- $\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$
- $\frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$
- $\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$
- $\frac{d}{dx}(\cot u) = -\csc^2 u \cdot \frac{du}{dx}$
- $\frac{d}{dx}(\sec u) = \sec u \tan u \cdot \frac{du}{dx}$
- $\frac{d}{dx}(\csc u) = -\csc u \cot u \cdot \frac{du}{dx}$

See examples 10, 11 done in class

Combination of different formulas

See examples 12, 13, 14, 15, 16 done in class

Application to tangent lines

See example 17 done in class

End of 3.5