Learning outcomes

After completing this section, you will inshaAllah be able to

- 1. find linear approximation of non-linear functions
- 2. find differential of a function
- 3. use differentials to approximate small changes or errors

Linear approximation

- Why do we need linear approximations?
 - See class explanation
- How to approximate?
 - Given a function y = f(x).
 - Equation of its tangent line at (a, f(a)) is

$$y - f(a) = f'(a)(x - a)$$

or

$$y = f(a) + f'(a)(x - a)$$

Main idea of linear approximation

"Near point x=a, the tangent line and the function f(x)

have approximately same graph"

See class explanation

Linear Approximation of f(x)

For values of x near x=a

$$f(x) \approx f(a) + f'(a)(x - a).$$

See examples 1, 2 done in class

Differential of a function

Given a function y = f(x).

The differential 'dy' of y is given by

$$dy = f'(x)dx$$

where dx denotes change in x

See class explanation

See example 3 done in class

Finding change Δy in y = f(x) corresponding to change $\Delta x = dx$ in x

Given a function y = f(x).

If x changes from x to x+dx then

$$\Delta y = f(x + \Delta x) - f(x) \tag{*}$$

See class explanation to see difference between dy and Δy

See example 4 done in class

Using differentials to approximate small change Δy in the function

- Note: For small $\Delta x = dx$ we have $\Delta y \approx dy$.
- Since finding dy is easy, it is a good idea to use dy to approximately find Δy .

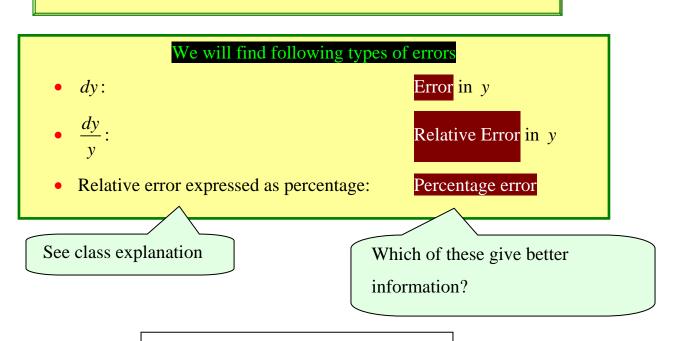
See example 5 done in class

Application of differentials in estimating the errors

When you make measurements "are you always exact?"

- Suppose we make a small error Δx in measuring x.
- This will obviously lead to an error Δy in y = f(x).
- As seen above, for a small change Δx , we have $\Delta y \approx dy$.

So we can use dy to estimate error Δy in y = f(x)



See example 6 done in class