Math 514 (092)
Midterm Take Home Exam
Due: Monday May 10.

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1. Choose the negative real axis to be the branch cut for $\log z$ and $\sqrt{z}$. Determine the branch cuts for $\ln\left(2+\sqrt{z+1}\right)$ that correspond to the two branches of $\sqrt{z}$.

2. Evaluate: $PV\int_{-\infty}^{\infty}\frac{\cos x}{1-x^3}\,dx$.

3. Use the definition to evaluate the Fourier transform and to verify the inverse formula for $f(t)=\frac{1}{t+1}$.

4. Find a solution that satisfies:

$$u_{xx}+y^2u_{yy}+yu_y=0,\qquad 0<x<1,\qquad 0<y<\infty,$$

$$u_x(0,y)=0,\ u(1,y)=H(1-y),\qquad 0<y<\infty.$$

   Comment on the behavior of the solution as $y\to 0$ and $y\to\infty$.

5. Solve

$$u_t=4u_{xx},\qquad 0<x<1,\ t>0,$$

$$u(0,t)=10,\ u(1,t)=0,\qquad t>0,$$

$$u(x,0)=0,\qquad 0<x<1.$$

6. Given that $\mathscr{L}\{J_0(ar)\}=\frac{1}{\sqrt{s^2+a^2}}$, show that $\mathscr{H}_1\{e^{-ar}\}=\frac{k}{(a^2+k^2)^{3/2}}$.
