Math101, Quiz # 2

Name:
ID #:

1. Find the values of m and n for which the function

$$f(x)=\begin{cases} x & \text{if } x\le 1\\ mx+n & \text{if } 1<x\le 2\\ x+2 & \text{if } x>2\end{cases}$$

For $f(x)$ to be cont.

Is continuous. $\Downarrow$

$\lim_{x\to1^-} f(x) = \lim_{x\to1} x = 1 = f(1) = 1 = \lim_{x\to1^+} f(x) = \lim_{x\to1} mx+n = m+n;\quad 1=m+n$

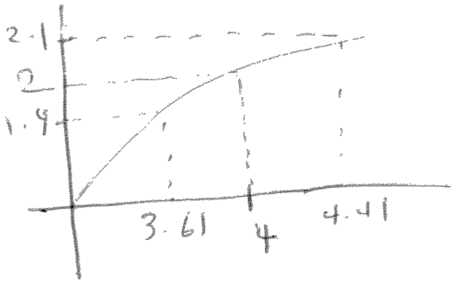
$\lim_{x\to2^-} f(x) = \lim_{x\to2} mx+n = 2m+n = f(2) = \lim_{x\to2^+} f(x) = \lim_{x\to2^+} x+2 = 4;\quad 4=2m+n$

$1=m+n \Rightarrow n=1-m \Rightarrow 4=2m+(1-m) \Rightarrow 4=m+1 \Rightarrow m=3$, $n=-2$

2. Show that the equation $e^{-x}=2-x$. has a root in the interval $(1, 2)$.
What is the name of the therom you used.

Let $f(x) = e^{-x}-2+x$ then the above equation has a root if $f(x)$ has a zero in $(1,2)$ but $f(x)$ is continuous and $f(1)=e^{-1}-2+1=e^{-1}-1<0$ and $f(2)=e^{-2}-2+2=e^{-2}>0$ by the intermediate value th° there is a root for $e^{-x}=2-x$ in $(1,2)$.

3. Let $f(x)=\sqrt{x}$. Find the largest value of δ such that $\left|\sqrt{x}-2\right|<0.1$ whenever $0<|x-4|<\delta$. (Hint: use the graph)



$2.1^2 = 2.1 \times 2.1$: 21, 42, 4.41

$1.9^2 = 1.9 \times 1.9 = 3.61$

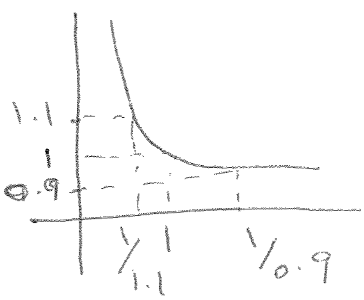
$\delta_1 = 4-3.61 = 0.39$

$\delta_2 = 4.41-4 = 0.41$

the largest value of δ is

$\delta = \min\{0.39, 0.41\}$
$= 0.39$.

4. Use Graph of $f(x)=\dfrac{1}{x}$ to find the largest number δ such that if $0<|x-1|<\delta$, then $|f(x)-1|<0.1$. (Show your work and write your answer in the simplest rational form)



$\frac{1}{0.9} = 1.1111 = \frac{10}{9}$

$\frac{1}{1.1} = 0.9091 = \frac{10}{11}$

$\Rightarrow \delta_1 = 1 - 0.9091 = 0.0909$

$\delta_2 = 1.1111 - 1 = 0.1111$

the largest $\delta = \min\{\delta_1, \delta_2\}$
$= 0.0909$.

smaller

$\left\{\frac{11}{11}-\frac{10}{11}\right\} = \frac{1}{11}$

$\frac{10}{9}-\frac{9}{9} = \frac{1}{9}$