Key Solution

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math - 106 Semester - 261 Quiz # 5

1. Evaluate the integral $\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx$,

$u = \sqrt[3]{8x^4} = 2x^{\frac{4}{3}}$ $\quad du = 2\left(\frac{4}{3}\right)x^{\frac{1}{3}}dx = \frac{8}{3}x^{\frac{1}{3}}dx$

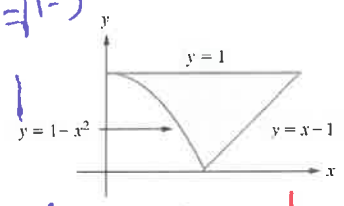
$I = \frac{3}{8}\int \frac{8}{3}\sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx = \frac{3}{8}\int e^u du = \frac{3}{8}e^u + c$

$I = \frac{3}{8}e^{\sqrt[3]{8x^4}} + c$

2. If $\int_1^2 f(x)dx = 5$ and $\int_3^1 f(x)dx = 2$, then $\int_2^3 f(x)dx =$

$\int_2^3 = \int_1^3 - \int_1^2 = -\int_3^1 - \int_1^2 = -2 - 5 = -7$

3. Find the area of the shaded region in the following figure is expressed as



$x = \sqrt{1-y}$ $\quad x = y+1$

$A = \int_0^1 1 - (1-x^2)\,dx + \int_1^2 1 - (x-1)\,dx$

$= \int_0^1 x^2 dx + \int_1^2 2 - x\,dx$

$= \frac{x^3}{3}\Big|_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$

$= \frac{1}{3} + \left[4 - \frac{4}{2} - \left(2 - \frac{1}{2}\right)\right]$

$= \frac{5}{6}$

using y-axis

$\int_0^1 y+1 - \sqrt{1-y}\, dy$

$F(x)$

$\frac{y^2}{2} + y - \frac{2}{3}(1-y)^{\frac{3}{2}}\Big|_0^1$

$F(1) = \frac{1}{2} + 1 - \frac{2}{3}(1-1)$

$= 1.5$

$F(0) = 0 + 0 - \frac{2}{3}$

$F(1) - F(0) = 1.5 + \frac{2}{3}$

$= 5/6$