Each question worth 10 points

King Fuhd University of Petroleum and Minerals
Department of Mathematical Sciences

**Math 101-13**
**First Exam , Semester 022**
Time:10:00-11:00 am, Wednesday, March 26, 2003

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Name :-------------------------------------------------ID # :-----------------------Section #----

Evaluate the following limits if they exist. But if they do not exist, give reasons

**Q1.** $\lim_{x\to 2}\left(\frac{x^2}{x-2}-\frac{4}{x-2}\right)$

$$= \lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(\cancel{x-2})(x+2)}{\cancel{x-2}}$$

$$= 4$$

**Q2.** $\lim_{x\to 0^-} [[x]] - x^2 = \lim_{x\to 0^-} [\![x]\!] - \lim_{x\to 0^-} x^2$

$$= -1 - 0 = -1$$

**Q3.** $\lim_{x\to -\infty} \frac{4x-3}{\sqrt{x^2+1}} = \lim_{x\to -\infty} \frac{4x-3/|x|}{\sqrt{x^2+1}/|x|}$

$$= \lim_{x\to -\infty} \frac{\frac{4x}{|x|} - \frac{3}{|x|}}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}} = \frac{-4-0}{\sqrt{0+1}} = -4$$

Q4 $\lim_{x\to 0} \csc 3x \tan 2x \cdot \frac{x}{x}$

$$= \lim_{x\to 0} \frac{3x}{3\sin 3x} \lim_{x\to 0} \frac{2\sin 2x}{2x} \cdot \lim_{x\to 0} \cos 2x$$

$$= \frac{2}{3} \lim_{x\to 0} \frac{3x}{\sin 3x} \lim_{x\to 0} \frac{\sin 2x}{2x} \lim_{x\to 0} \cos 2x$$

$$= \frac{2}{3} (1) \quad (1) \quad (1)$$

$$= 2/3$$

Q5 $\lim_{x\to 0} \frac{|x|}{\sqrt{x^4+4x^2+7}}$ ( Use squeezing Th.)

$$\sqrt{x^4+4x^2+7} \geq \sqrt{7}$$

$$0 < \frac{1}{\sqrt{x^4+4x^2+7}} \leq \frac{1}{\sqrt{7}} < 1$$

multiply by $|x|$

$$0 |\cdot x| < \frac{|x|}{\sqrt{x^4+4x^2+7}} < 1 \cdot |x|$$

but $\lim_{x\to 0} 0 = 0$

$\lim_{x\to 0} |x| = 0$

using squeezing th

$$\lim_{x\to 0} \frac{|x|}{\sqrt{x^4+4x^2+7}} = 0$$

Q6. Using Intermediate value theorem show that the equation $x^3+1=0$ has a solution between -2 and 2?

$f(-2) = -8+1 = -7$

$f(2) = 9$

Since $x^3+1$ continuous function and $f(-2)$ & $f(2)$ have opposite sign then $\exists c \in (-2, 2)$ such that

$f(c) = 0$

**Q7.** Let $f(x)=\begin{cases} c & if \quad x=-3 \\ \dfrac{9-x^2}{4-\sqrt{x^2+7}} & if \quad |x|<3 \\ d & if \quad x=3 \end{cases}$ find the values of $c$ and $d$ such that $f(x)$ is continuous on [-3,3]

$$f(-3)=c=\lim_{x\to -3^+} f(x)=\lim_{x\to -3^+}\frac{9-x^2}{4-\sqrt{x^2+7}}\cdot\frac{4+\sqrt{x^2+7}}{4+\sqrt{x^2+7}}$$

$$=\lim_{x\to -3^+}\frac{(9-x^2)(4+\sqrt{x^2+7})}{16-x^2-7}=4+\sqrt{9+7}=8=c$$

$$f(3)=d=\lim_{x\to 3^-} f(x)=8=d.$$

**Q8.** Find the instantaneous velocity for $s(t)=\sqrt{t+1}$ at a general point $t=t_0$

$$\lim_{t\to t_0}\frac{f(t)-f(t_0)}{t-t_0}=\lim_{t\to t_0}\frac{\sqrt{t+1}-\sqrt{t_0+1}}{t-t_0}\left(\frac{\sqrt{t+1}+\sqrt{t_0+1}}{\sqrt{t+1}+\sqrt{t_0+1}}\right)$$

$$=\lim_{t\to t_0}\frac{t+1-t_0-1}{t-t_0}\cdot\frac{1}{\sqrt{t+1}+\sqrt{t_0+1}}=\frac{1}{\sqrt{t_0+1}+\sqrt{t_0+1}}$$

$$=\frac{1}{2\sqrt{t_0+1}}$$