⑤ $\min\ -2x_1 + x_2 - x_3$

s.t $x_1 + x_2 + x_3 \leq 6 \rightarrow x_1 + x_2 + x_3 + x_4 = 6$

$-x_1 + 2x_2 \leq 4 \rightarrow -x_1 + 2x_2 + x_5 = 4$

$x_i \geq 0 \qquad x_i \geq 0$

x_4	①	1	1	1	0	6
x_5	−1	2	0	0	1	4
	−2	1	−1	0	0	0

x_4 will leave the base

x_1 " enter the base

x_1	1	1	1	1	0	6
x_5	0	3	1	1	1	10
	0	3	1	2	0	12

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$z = -12$

⑦ $\min\ -3x_1 - 2x_2$

Phase 01

$\min\ y_1 + y_2$

s.t $3x_1 + 4x_2 \leq 5 \rightarrow 3x_1 + 4x_2 + x_3 = 5$

$2x_1 + x_3 \geq 2 \rightarrow 2x_1 + x_3 - x_4 + y_1 = 2$

$-x_1 + x_2 \geq 1 \rightarrow -x_1 + x_2 - x_5 + y_2 = 1$

$x_i \geq 0$

Phase 01

x_3	3	4	1	0	0	0	0	5
y_1	2	1	0	−1	0	1	0	2
y_2	−1	1	0	0	−1	0	1	1
	0	0	0	0	0	1	1	0

x_3	3	4	1	0	0	0	0	5
y_1	2	1	0	−1	0	1	0	2
y_2	−1	1	0	0	−1	0	1	1
	−1	(−2)	0	1	1	0	0	−3

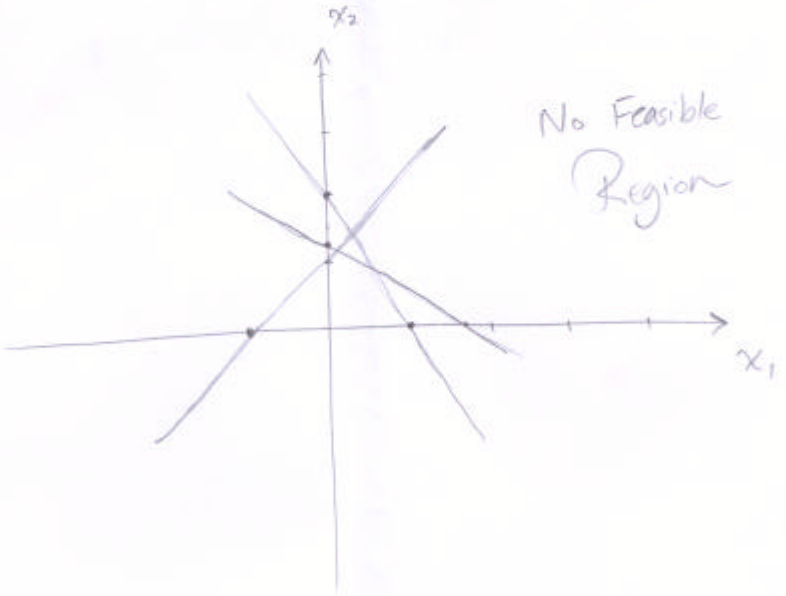
x_2 will enter

y_2 will leave

x_2	2	0	1	0	4	0	−4	1	x_1 will enter
y_1	3	0	0	−1	1	1	−1	1	x_3 will leave
x_3	−1	1	0	0	−1	0	1	1	
	(3)	0	0	1	−1	0	2	−1	

x_1	1	0	1/7	0	4/7	0	−4/7	1/7
y_1	0	0	−3/7	−1	−5/7	1	5/7	4/7
x_2	0	1	1/7	0	−3/7	0	3/7	8/7
	0	0	3/7	1	5/7	0	1	−4/7

Since $r_j \geq 0$ but $z \neq 0$ $\Rightarrow$ the feasible region is <u>empty</u>.



No Feasible Region

(14) $\min \; z = -2x_1 + 3x_2 - 4x_3$

$s.t \quad x_1 - 2x_2 + 4x_3 \leq 15 \quad \rightarrow \quad x_1 - 2x_2 + 4x_3 + x_4 = 15$

$2x_1 - x_2 + 5x_3 \leq 10 \quad \rightarrow \quad 2x_1 - x_2 + 5x_3 + x_5 = 10$

$x_i \geq 0 \qquad x_i \geq 0$

x_4	1	-2	4	1	0	15
x_5	2	-1	5	0	1	10
	(-2)	3	-4	0	0	0

Since $r_1 = -2$ is the first negative, x_1 will enter the base.

x_5 will leave the base.

x_4	0	-3/2	3/2	1	-1/2	10
x_1	1	-1/2	5/2	0	1/2	5
	0	2	1	0	1	10

$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$

$z = -10$

(22) $\max \; 5x_1 + 6x_2 + 9x_3 + 8x_4$

s.t $x_1 + 2x_2 + 3x_3 + x_4 \leq 5$

$x_1 + x_2 + 2x_3 + 3x_4 \leq 3$

$x_i \geq 0$

$\min \; -5x_1 - 6x_2 - 9x_3 - 8x_4$

s.t $x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 5$

$x_1 + x_2 + 2x_3 + 3x_4 + x_6 = 3$

$$d^T = c_B^T B^{-1} = [0\ 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0\ 0]$$

$$r_N^T = c_N^T - d^T N = c_N^T = [-5\ -6\ -9\ -8]$$

x_3 will enter the base.

$$a_3^* = B^{-1} a_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$r_3 = c_3 - d^T a_3 = -9 - 0 = -9$$

	B^{-1}		b	a_3	
x_5	1	0	5	3	
x_6	0	1	3	(2)	x_6 will leave the base
	0	0	0	-9	

x_5	1	$-\frac{3}{2}$	$\frac{1}{2}$	0
x_3	0	$\frac{1}{2}$	$\frac{1}{2}$	1
	0	$\frac{9}{2}$	$\frac{9}{2}$	0

$$d^T = C_B^T B^{-1} = \begin{bmatrix} 0 & -9 \end{bmatrix} \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{9}{2} \end{bmatrix}$$

$$r_1 = c_1 - d^T a_1 = -5 - \begin{bmatrix} 0 & -\frac{9}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + \frac{9}{2} = -\frac{1}{2}$$

$$r_2 = c_2 - d^T a_2 = -6 - \begin{bmatrix} 0 & -\frac{9}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -6 + \frac{9}{2} = -\frac{3}{2}$$

$$r_4 = c_4 - d^T a_4 = -8 - \begin{bmatrix} 0 & -\frac{9}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -8 + \frac{27}{2} = \frac{11}{2}$$

$$r_6 = c_6 - d^T a_6 = 0 - \begin{bmatrix} 0 & -\frac{9}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + \frac{9}{2} = \frac{9}{2}$$

x_2 will enter the base

$$a_2^* = B^{-1} a_2 = \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

x_5	1	$-\frac{3}{2}$	$1/2$	$\frac{1}{2}$
x_3	0	$1/2$	$3/2$	$\frac{1}{2}$
	0	$9/2$	$\frac{27}{2}$	$-\frac{3}{2}$

x_5 will leave the base

x_2	2	−3	1	1
x_3	−1	2	1	0
	3	0	15	0

$$d^T = C_B^T B^{-1} = \begin{bmatrix} -6 & -9 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \end{bmatrix}$$

$$r_1 = c_1 - d^T a_1 = -5 - \begin{bmatrix} -3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + 3 = -2$$

$$r_4 = c_4 - d^T a_4 = -8 - \begin{bmatrix} -3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = -8 + 3 = -5$$

$$r_5 = c_5 - d^T a_5 = 0 - \begin{bmatrix} -3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 + 3 = 3$$

$$r_6 = c_6 - d^T a_6 = 0 - \begin{bmatrix} -3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + 0 = 0$$

x_4 will enter the base

$$a_4^* = B^{-1} a_4 = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

x_2	2	−3	1	−7
x_3	−1	2	1	(5)
	3	0	15	−5

x_3 will leave the base.

x_2	$3/5$	$-1/5$	$17/5$	0
x_4	$-1/5$	$2/5$	$1/5$	1
	2	2	16	0

$$d^T = C_B^T B^{-1} = \begin{bmatrix} -6 & -8 \end{bmatrix} \begin{bmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{bmatrix} = \begin{bmatrix} -2 & -2 \end{bmatrix}$$

$$r_1 = c_1 - d^T a_1 = -5 - \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + 4 = -1$$

$$r_3 = c_3 - d^T a_3 = -9 - \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = -9 + 11 = 1$$

$$r_5 = c_5 - d^T a_5 = 0 - \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2$$

$$r_6 = c_6 - d^T a_6 = 0 - \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2$$

x_1 will enter the base

$$a_1^* = B^{-1} a_1 = \begin{bmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2/5 \\ 1/5 \end{bmatrix}$$

x_2	$3/5$	$-1/5$	$13/5$	$2/5$
x_4	$-1/5$	$3/5$	$1/5$	$(1/5)$
	2	2	16	-1

x_4 will leave the base.

	B^{-1}		b	
y_2	1	-1	2	0
x_1	-1	2	1	1
	1	4	17	0

$$d^T = c_B^T B^{-1} = \begin{bmatrix} -6 & -5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \end{bmatrix}$$

$$r_3 = c_3 - d^T a_3 = -9 - \begin{bmatrix} -1 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = -9 + 11 = 2$$

$$r_4 = c_4 - d^T a_4 = -8 - \begin{bmatrix} -1 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = -8 + 13 = 5$$

$$r_5 = c_5 - d^T a_5 = 0 - \begin{bmatrix} -1 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 + 1 = 1$$

$$r_6 = c_6 - d^T a_6 = 0 - \begin{bmatrix} -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + 4 = 4$$

Since all $r_N \geq 0 \Rightarrow$ OBFS $\rightarrow \underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ and $z = -17$