②

$$x_1 + 2x_2 + x_3 = 7$$
$$2x_1 - x_2 + 2x_3 = 6$$
$$x_1 + x_2 + 3x_3 = 12$$

1)

①	2	1	7
2	-1	2	6
1	1	3	12

2)

1	2	1	7
0	-5	0	-8
0	-1	2	5

3)

1	2	1	7
0	1	0	8/5
0	-1	2	5

4)

1	0	1	19/5
0	1	0	8/5
0	0	②	33/5

5)

1	0	1	19/5
0	1	0	8/5
0	0	1	55/10

6)

1	0	0	5/10
0	1	0	8/5
0	0	1	33/10

$$x_1 = \frac{5}{10}$$

$$x_2 = \frac{8}{5}$$

$$x_3 = \frac{33}{10}$$

(8) max $-x_1 + x_2$ → min $z = x_1 - x_2$

(a) s.t $x_1 - x_2 \leq 2$ → $x_1 - x_2 + x_3 = 2$

$x_1 + x_2 \leq 6$ → $x_1 + x_2 + x_4 = 6$

$x_1 \geq 0, x_2 \geq 0$ → $x_i \geq 0$

x_3	1	-1	1	0	2
x_4	1	①	0	1	6
	1	-1	0	0	0

x_2 will enter the base

x_4 will leave the base

x_3	2	0	1	1	9
x_2	1	1	0	1	6
	2	0	0	1	⑥ ← $-z$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

(b)

$(0,6)$
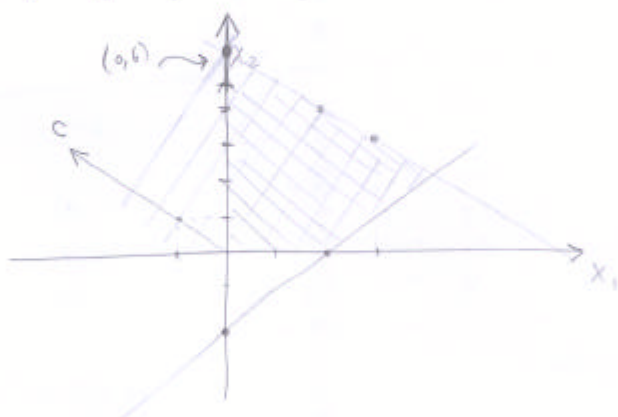
(c) max $x_1 + x_2$ → min $z = -x_1 - x_2$

St $-2x_1 + x_2 \leq 1$ → St $-2x_1 + x_1 + x_3 = 1$

$x_1 - x_2 \leq 1$ → $x_1 - x_2 + x_4 = 1$

$x_1 \geq 0$ $x_2 \geq 0$

x_3	-2	1	1	0	1
x_4	①	-1	0	1	1
	-1	-1	0	0	0

x_1 will enter the base

x_4 will leave the base

x_3	0	(-1)	1	2	3
x_1	1	(-1)	0	1	1
	0	(-2)	0	1	1

Unbounded

C

Unbounded
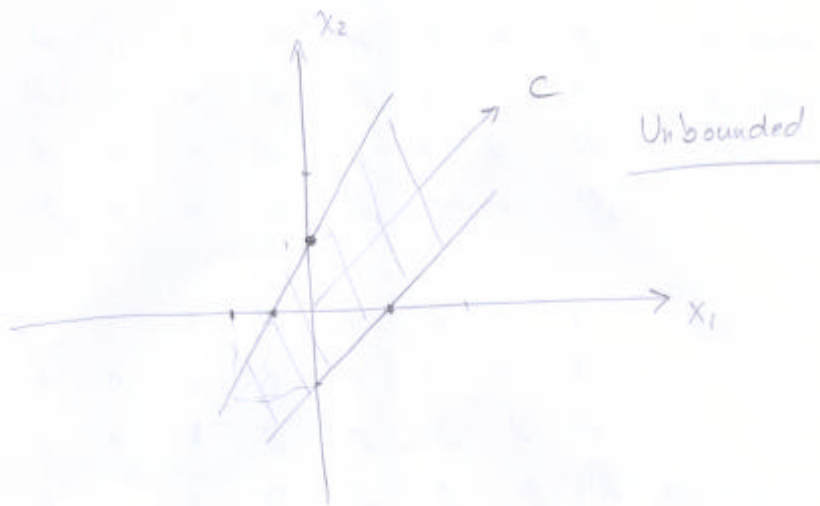
(10)   max $2x_1 + 4x_2 + x_3 + x_4$ →   min $-2x_1 - 4x_2 - x_3 - x_6$
                                              s.t
       st $x_1 + 3x_2$ $+x_4 \le 4$ → $x_1 + 3x_2$ $+ x_4 + x_5 = 4$
             $2x_1 + x_2$ ≤ 3 → $2x_1 + x_2$ $+ x_6 = 3$
                   $x_2 + 4x_3 + x_4 \le 3$ → $x_2 + 4x_3 + x_6 + x_7 = 3$

             $x_i \ge 0$ $x_i \ge 0$

x_5	1	③	0	1	1	0	0	4
x_6	2	1	0	0	0	1	0	3
x_7	0	①	4	1	0	0	1	3
	-2	-4	-1	-1	0	0	0	0

x_2 will enter the base

x_5 will leave the base

x_2	1/3	1	0	1/3	1/3	0	0	4/3
x_6	5/3	0	0	-1/3	-1/3	1	0	5/3
x_7	-1/3	0	4	2/3	-1/3	0	1	5/3
	-2/3	0	-1	1/3	4/3	0	0	16/3

x_3 will enter the base

x_7 will leave the base

x_2	$1/5$	1	0	$1/5$	$1/20$	0	0	$4/5$	x_1 enter
x_6	$(5/3)$	0	0	$-1/9$	$-1/8$	1	0	$5/3$	x_6 leave
x_3	$-1/12$	0	1	$2/12$	$-1/12$	0	$1/6$	$5/12$	
	$-9/12$	0	0	$1/2$	$15/12$	0	$1/6$	$69/12$	

x_2	0	1	0	$6/15$	$4/15$	$-1/15$	0	1
x_1	1	0	0	$-1/5$	$-1/5$	$3/15$	0	1
x_3	0	0	1	$9/10$	$-1/10$	$1/20$	$1/6$	$1/2$
	0	0	0	$17/20$	$1/10$	$9/20$	$3/15$	$(13/2)$ $\leftarrow -z$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1/2 \\ 0 \end{bmatrix}$$