HW#1

3. Adam Smith has $18,000 to invest in three types of stocks: low-risk, medium-risk, and high-risk. He wants to invest this money in a way that maximizes his total profit subject to three constraints that (theoretically) offer some protection against large losses. He will invest at most $2000 more in low-risk stocks than he invests in medium-risk stocks. He will invest at most $8000 in high-risk stocks, and no more than $14,000 in medium- and high-risk stocks. The expected returns are 7% for low-risk stocks, 9% for medium-risk stocks, and 11% for high-risk stocks. How much money should Adam invest in each type of stock, and how much return can he expect?

5. A family has a home in the country that has 4 acres of lawn. During the growing season they spend 6 person-hours and $15 per acre to maintain the lawn. They are considering plowing part of the land to grow crops for sale at the local farmers' market and want to estimate how profitable this would be. Table 5 contains their estimates of the labor and investment required and the gross sales expected for each acre. The last row of Table 5 gives limitations on the time (hours) they can spend and the money (dollars) they can invest in such costs as fertilizer, pesticides, and water. Determine how much land they should use for each crop to maximize profit. Any land that is not planted for crops to sell must still be maintained as lawn.

Table 5

Crop	Labor (person-hr)	Expenses ($)	Gross sales ($)
Tomatoes	90	150	750
Beans	18	40	150
Peas	27	30	160
Corn	10	25	50
Carrots	30	55	180
Constraints	300	400	

7. A plant manager is planning a week's production to manufacture five products A, B, C, D, and E. Products can be produced in any combination, except that the plant has already accepted an order for 20 units of product C and 30 units of product D; so at least these amounts of the two products must be made. The manufacture of each of the five products requires time on three machines: M_1, M_2, and M_3. Each machine is available for 80 hours per week. Table 6 gives the processing time (minutes) and the selling price (dollars) for one unit of each product.

The costs of operating the machines are $9 per hour each for machines 1 and 2 and $12 per hour for machine 3. Materials costs are $2 per unit for products A, B, and C and $1 per unit for products D and E.

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Table 6

Product	Required time on machine (min) M_1	M_2	M_3	Selling price ($)
A	15	8	6	12.00
B	8	10	9	11.00
C	8	12	10	12.00
D	12	4	12	10.50
E	9	6	0	6.00

(a) Introduce appropriate variables and write the objective function for the profit. The profit is the total selling price of all units, minus the cost of materials used in making the units, and minus the cost of operating the machines. Assume there is no standby cost for a machine not being used.
(b) Write the constraint inequalities.

13. A paint manufacturer uses three minerals to provide four chemicals required in its paint. The composition of the paint must be at least 4% of chemical A, 3% of chemical B, 30% of chemical C, and 16% of chemical D. Table 11 gives the relevant compositions of the

Table 11

Mineral	Chemical A (%)	B (%)	C (%)	D (%)	Cost ($/lb)
Mineral 1	3	5	35	24	3.50
Mineral 2	7	8	32	12	2.50
Mineral 3	9	1	27	15	3.00

minerals and the unit costs. Because mineral 2 causes an undesirable color when use excess, no more than 1% of the total mineral content of the paint can be miner Determine the mixture of minerals that will provide the necessary composition chemicals at the least cost.

(Suggestion: The least confusing way to analyze a problem involving percentages i mix a fixed amount of the product, say 100 pounds. Thus, 100 pounds of paint wo have to contain 4 pounds of chemical A, 3 pounds of chemical B, 30 pounds of chem C, and 16 pounds of chemical D.)