Q1. Find y', if $y = (\sec x)^{\log x}$

$\ln y = \log x \ln \sec x$; $\frac{y'}{y} = \frac{1}{x \ln 10} \ln \sec x + \log x \frac{\sec x \tan x}{\sec x}$

$y' = (\sec x)^{\log x}\left(\frac{\ln \sec x}{x \ln 10} + \log x \tan x\right)$

Q2. If $x + \sin^{-1}(\pi) = \tan^{-1}(\pi - y)$, find y', then prove that $y'' = 2y'(\pi - y)$

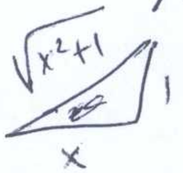
Q3. Find the nth derivative of $y = 2^{3x}$

$y' = 2^{3x} 3 \ln 2$ $y'' = 2^{3x} 3 \ln 2 . 3 \ln 2 = 2^{3x} (3 \ln 2)^2$

$y^{(n)} = 2^{3x} (3 \ln 2)^n$

Q4. Prove that $\frac{d \cot^{-1} x}{dx} = \frac{-1}{1 + x^2}$.

$y = \cot^{-1} x$ $\cot y = x$ $-y' \csc^2 y = 1$ $y' = \frac{-1}{\csc^2 \cot^{-1} x}$



$y' = \frac{-1}{x^2 + 1}$

Q5. Prove that $f(x) = x^3 + 2x - 1$ has an inverse and then find the slope of the tangent line to the graph of $f^{-1}(x)$ at $x = 2$.

$f'(x) = 3x^2 + 2 > 0$ Increasing

1-1 $\Rightarrow$ has inverse

$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{5}$

$f^{-1}(2) = d \Rightarrow f(d) = 2$ $d^2 + 2d - 1 = 2 \Rightarrow \underline{d = 1}$

Q6. At what point(s) is (are) the tangent line to the graph of $y = 10^x + x \ln 100$ is perpendicular to the line $y \ln 1000 + x + 2 = 0$. Show your final answer.

$y' = 10^x \ln 10 + \ln 100$ ~~m~~ $y = -\frac{x}{\ln 1000} + \cdots$

$10^x \ln 10 + \ln 100 = \ln 1000$ $m = \ln 1000$

$x = 0$

Q7. A spherical snowball is melting at the rate of $2\pi\ cm^3/\sec$. How fast is the raduis changing when it is $3\ cm$. $V = (4/3)\pi r^3$.

$V' = \frac{4}{3}\pi 3r \dot{r} \Rightarrow \dot{r} = \frac{-1}{18}$ cm/sec

Q8. State and then prove the Quotient Derivative Rule (Division Rule).

See book