1. a. Evaluate $\lim_{x\to-\infty} \dfrac{\sqrt{3x^2-1}}{4x-5}$

$$\lim_{x\to-\infty} \frac{\sqrt{\frac{3x^2}{x^2}-\frac{1}{x^2}}}{4\frac{x}{|x|}-\frac{5}{|x|}} = \lim_{x\to\infty} \frac{\sqrt{3-\frac{1}{x^2}}}{-4-\frac{5}{|x|}}$$

$$= \frac{-\sqrt{3}}{4}$$

b. Find y' if $y = (\ln x)^{\cot x}$

$$\ln y = \cot x \ \ln(\ln x)$$

$$\frac{y'}{y} = -\csc^2 x \ln(\ln x) + \frac{\cot x}{x \ln x}$$

$$y' = (\ln x)^{\cot x}\left[\frac{\cot x}{x\ln x} - \csc^2 x \ln(\ln x)\right]$$

20. a. Evaluate $\lim_{x\to 0}(\cos x)^{1/x}$ 1^{∞}

$y = \lim_{x\to 0}(\cos x)^{\frac{1}{x}}$

$\ln y = \lim_{x\to 0} \frac{1}{x} \ln(\cos x)$

$= \lim_{x\to 0} \frac{\ln(\cos x)}{x} \left(\frac{0}{0}\right)$

l'Hopital

$= \lim_{x\to 0} \frac{-\tan x}{1} = 0$

$\ln y = 0$

$y = e^0 = 1$

b. Find y' if $xy = 13^{2x}$

$y + xy' = 13^{2x}\, 2\ln 13$

$y' = \frac{(13^{2x})(2\ln 13) - y}{x}$

19. Sketch the Graph of the function $f(x) = 2 + \frac{5}{2}x^{2/3}$

Domain $\mathbb{R}$

Symmetry around the y-axis $f(-x) = 2 + \frac{5}{2}(-x)^{2/3} = f(x)$

$\lim_{x\to\infty} f(x) = \infty = \lim_{x\to-\infty} f(x)$

No Horizontal or vertical asymptotes

x-intercept $f(x) = 0 = 2 + \frac{5}{2}x^{2/3}$

$\Rightarrow \sqrt[3]{x^2} = \frac{-4}{5} \Rightarrow x^2 = -\left(\frac{4}{5}\right)^3$

No x-intercept.

y-intercept if $x=0$

$y = 2$ $(0,2)$

f +

$f'(x) = \frac{5}{3}x^{-1/3} = \frac{5}{3\sqrt[3]{x}}$

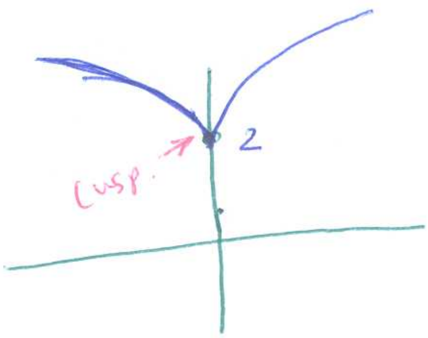
f'

Relative min

$f''(x) = \frac{-5}{9\sqrt[3]{x^4}}$

f'' 0

$\left.\begin{array}{l}\lim_{x\to0^-} f'(x) = \infty \\ \lim_{x\to0^+} f'(x) = -\infty\end{array}\right\}$ cusp at $x=0$



18. Discuss the motion of a particle moves in straight line according to the position function $s(t) = t^3 - 6t^2 + 9t$

$v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-1)(t-3)$

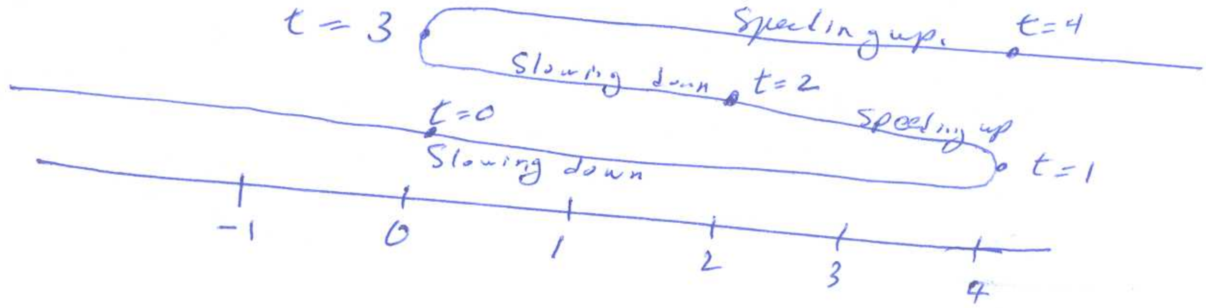
$a(t) = 6t - 12 = 6(t-2)$

$t = 1,\ t = 3$

$t = 2$

t	0	1	2	3	4
$s(t)$	0	4	2	0	4
$v(t)$	9	0	-3	0	9
$a(t)$	-12	-6	0	6	12

$(-\infty, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
Slowing down	Speeding up	Slowing down	Speeding up
Right	left	left	Right



17. Sketch the graph of f that satisfies the condition

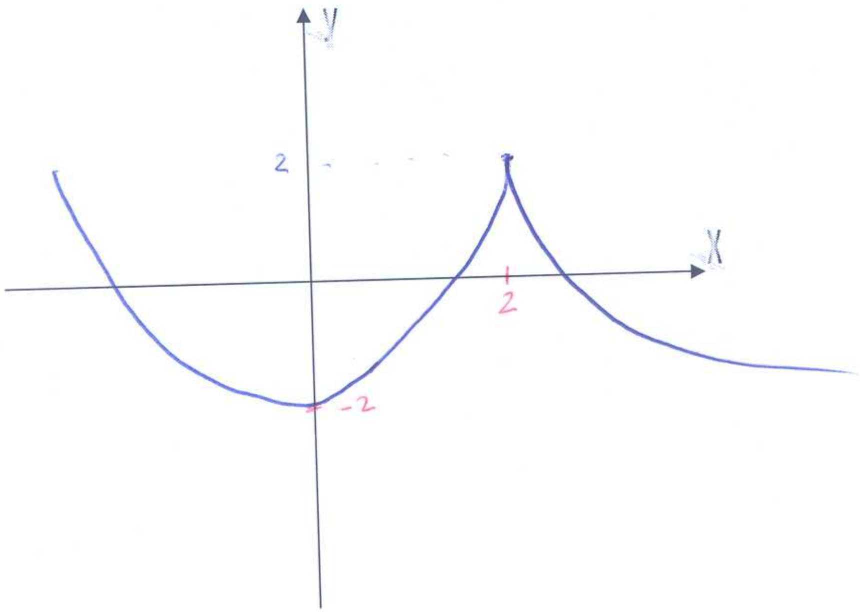
$$f(0) = -2 \qquad f(2) = 2$$

$$f'(0) = 0 \qquad f'(2) \text{ undefined}$$

$$f'(x) > 0 \quad \text{if} \quad 0 < x < 2$$

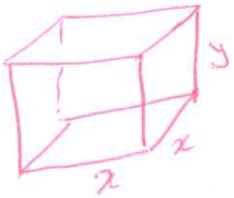
$$f'(x) < 0 \quad \text{if} \quad x < 0 \text{ or } x > 2$$

$$f''(x) > 0 \quad \text{for} \quad x < 2 \text{ or } x > 2$$



15. A closed rectangular box with a square base is to have a volume 20,000 cm^3. The material for the bottom of the box will cost 3 S. R. per cm^2, and the material for the sides and the top of the box will cost 2 S. R. per cm^2. Find the dimensions that will minimize the cost of the material.

$V = x^2 y = 20{,}000$



$[1, 50]$

$$\text{Cost} = C(x,y) = \underbrace{3(x^2)}_{\text{bottom}} + \underbrace{2(x^2)}_{\text{top}} + \underbrace{2(4)xy}_{4\,\text{sides}}$$

Cost

$$C(x) = 5x^2 + 8\cancel{x}\frac{20000}{x^{\cancel{2}}} = 5x^2 + \frac{8(20\,000)}{x}$$

$$\frac{dC}{dx} = C' = 10x - \frac{160000}{x^2} = 0$$

$$x^3 = \frac{160\,000}{10} \Rightarrow x = \sqrt[3]{16000} = 10\sqrt[3]{16} = 20\sqrt[3]{2} \text{ cm}$$

$$y = \frac{20{,}000}{400\sqrt[3]{4}} = \frac{100}{2\sqrt[3]{4}} = \frac{50}{\sqrt[3]{4}} \text{ cm}$$

16. Evaluate $\lim\limits_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{2e^x - 2} \right)$ $(\infty - \infty)$

$$= \lim_{x \to 0} \frac{2e^x - 2 - x^2}{2x^2 e^x - 2x^2} \left(\frac{0}{0}\right) = \lim_{x \to 0} \frac{2e^x - 2x}{4xe^x + 2x^2 e^x - 4x}$$

$$= \frac{2}{0} = \infty \qquad x \to 0^+$$

$$-\infty \qquad x \to 0^-$$

14. State The Mean Value theorem and verify that the function $f(x) = x^2 + 2x$ satisfies the hypotheses of The Mean Value theorem on the interval [-1, 1]. Then find a number c that satisfy its conclusion on this interval.

If $f(x)$ cont. at $[a,b]$ and diff at (a,b)

then there is $c \in (a,b)$ s.t.

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$f(x)$ is continuous and diff because it is a polynomial

$\Rightarrow$ satisfy MVT.

$f'(x) = 2x + 2$ $\qquad f'(c) = 2c + 2$

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{1+2-(1-2)}{2} = \frac{4}{2} = 2$$

$f'(c) = 2 \Rightarrow 2c + \cancel{2} = \cancel{2}$

$\underline{c = 0}$

12. If $x^2 + y^2 = 1$ show that $y'' = \dfrac{-1}{y^3}$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

$$y'' = \frac{-1 \cdot y - (-x)\,y'}{y^2} = \frac{xy' - y}{y^2}$$

$$= \frac{x\left(-\frac{x}{y}\right) - y}{y^2}$$

$$= \frac{-\frac{x^2}{y} - \frac{y^2}{y}}{y^2}$$

$$= \frac{-\overbrace{(x^2+y^2)}^{1}}{y^3} = \frac{-1}{y^3}$$

13. Find the absolute extrema for the function $f(x) = x^4 - 2x^2 - 3$ in $[-3,3]$

$$f'(x) = 4x^3 - 4x = 0 \qquad 4x(x^2-1) = 0$$

$$x = 0, \quad x = 1, \quad x = -1$$

x	$f(x)$
-3	60
0	-3
1	-4
-1	-4
3	60

abs max at $x = 3$ & $x = -3$

abs min at $x = 1$ & -1

10. Evaluate $\lim_{x\to 0} \frac{\cos^{-1}(0.5-x)-\cos^{-1}0.5}{x}$ $\left(\frac{\cos^{-1}0.5 - \cos^{-1}0.5}{0} = \frac{0}{0} \right)$

L'Hopital

$$\lim_{x\to 0} \frac{\frac{-1}{\sqrt{1-(0.5-x)^2}} - 0}{1} = \frac{-1}{\sqrt{1-(0.5)^2}}$$

$$= \frac{-1}{\sqrt{1-0.25}} = \frac{-1}{\sqrt{0.75}}$$

11. Evaluate $\lim_{t\to 0} \frac{(\frac{1}{3})^{\sin t}-1}{t}$ $\left(\frac{(\frac{1}{3})^0 - 1}{0} = \frac{0}{0} \right)$

L'Hopital

$$\lim_{t\to 0} \frac{(\frac{1}{3})^{\sin t} \cdot \ln\frac{1}{3}\ \cos t}{1} = 1 \cdot \ln(\tfrac{1}{3}) \cdot 1$$

$$= \ln\frac{1}{3}$$

$$= -\ln 3$$

8. Find y' if $y=\sqrt[3]{x}^{\sqrt{x}}$

$$\ln y = \sqrt{x}\ \ln \sqrt[3]{x}$$

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}} \ln\sqrt[3]{x} + \frac{\sqrt{x}}{\sqrt[3]{x}} \cdot \frac{1}{3\sqrt[3]{x^2}}$$

$$y' = \sqrt[3]{x}^{\sqrt{x}}\left[\frac{\ln\sqrt[3]{x}}{2\sqrt{x}} + \frac{\sqrt{x}}{3\ x}\right]$$

$$= \sqrt[3]{x}^{\sqrt{x}}\left[\frac{3(\ln\sqrt[3]{x})}{3(2\sqrt{x})} + \frac{2}{2(3\sqrt{x})}\right] = \sqrt[3]{x}^{\sqrt{x}}\left[\frac{2+3\ln\sqrt[3]{x}}{6\sqrt{x}}\right]$$

9. Show that $\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$

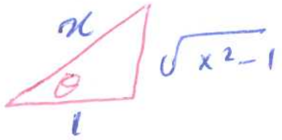
Let $y = \sec^{-1} x$ then $x = \sec y$

$1 = \sec y \tan y \ y'$

$$y' = \frac{1}{\sec y \tan y} = \frac{1}{\sec\sec^{-1}x\ \tan\sec^{-1}x}$$

Now $\sec\sec^{-1}x = |x|$ (cos even function)

$\tan\sec^{-1}x = \sqrt{x^2-1}$ Let $\theta = \sec^{-1}x$ $\boxed{\sec\theta = x}$



$\tan\theta = \sqrt{x^2-1}$

$$\Rightarrow y' = \frac{1}{|x|\sqrt{x^2-1}} \quad \square$$

6. Use Newton method to estimate $\sqrt[3]{8.01}$ with $x_0 = 2$ (one iteration)

Let $f(x) = x^3 - 8.01$

the root of $f(x)$ is $\sqrt[3]{8.01}$

$$X_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad f'(x) = 3x^2$$

$$= 2 - \frac{8 - 8.01}{3(4)} = 2 - \frac{-0.01}{12}$$

$$= \boxed{2 + \frac{0.01}{12}}$$

$$\approx \underline{2.0009}$$

7. Use Local linear approximation to estimate $\sqrt[3]{8.01}$ with $x_0 = 8$

Let $f(x) = \sqrt[3]{x}$ $\qquad f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$$l(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$= \sqrt[3]{8} + \frac{1}{3\sqrt[3]{64}} (x - 8)$$

$$l(x) = 2 + \frac{(x-8)}{12}$$

$$= \boxed{2 + \boxed{\frac{0.01}{12}}} \approx \underline{2.0009}$$

4. Find the equation of tangent line to the graph of the equation $x^2 + 2xy - y^2 + x = 2$ at the point $(1,2)$

$$\frac{y-2}{x-1} = \text{Slope} \qquad y'\Big|_{(1,2)}$$

$$2x + 2(y + xy') - 2yy' + 1 = 0$$

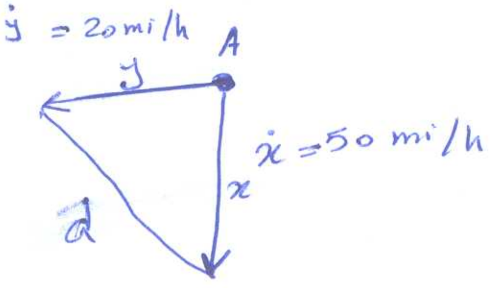
$$2x + 2y + 1 = 2yy' - 2xy'$$

$$y' = \frac{2x+2y+1}{2y-2x}\Bigg|_{(1,2)} = \frac{2+4+1}{4-2} = \frac{7}{2} \text{ Slope}$$

$$y - 2 = \frac{7}{2}(x-1) \quad \text{Equation of the tangent line}$$

5. Two boats start moving from the same point. One sailing south at 50 mi/h and the other sailing west at 20 mi/h. At what rate is the distance between the boats increasing one hour later?

$\dot{d}$?



$$x^2 + y^2 = d^2$$

$$(50)^2 + (20)^2 = d^2$$

$$2500 + 400$$

$$d = \sqrt{2900} = 10\sqrt{29} \text{ mi}$$

$$2x\dot{x} + 2y\dot{y} = 2d\dot{d}$$

$$\dot{d} = \frac{x\dot{x} + y\dot{y}}{d} = \frac{50(50) + 20(20)}{10\sqrt{29}} = \frac{2900}{1\cancel{0}\sqrt{29}}$$

$$= 10\sqrt{29} \text{ mi/hr}$$

2. Show that $f(x)=\sqrt{4-x^2}$ is continuous on the interval $[-2, \quad 2]$

Let $c \in (-2,2)$ then $\lim_{x \to c} f(x) = \sqrt{4-c^2} = f(c)$

$= \sqrt{\lim_{x \to c}(4-x^2)} = \sqrt{4-c^2} = f(c)$

$\lim_{x \to -2^+} f(x) = \sqrt{0^+} = 0 = f(-2)$

$\lim_{x \to 2^-} f(x) = \sqrt{0^+} = 0 = f(2)$

3. Let $f(x)=\begin{cases} 2-x & x<1 \\ \dfrac{1}{2-x} & x \geq 1 \end{cases}$, Find $f'_-(1)$ and $f'_+(1)$ is $f'(1)$ exists.

$f'_-(1) = (2-x)'\Big|_{x=1} = -1$

$f'_+(1) = \left(\frac{1}{2-x}\right)'\Big|_{x=1} = \frac{+1}{(2-1)^2} = +1$

$(2-x)^{-1}$

$+1(2-x)^{-2}$

$f'(1)$ Does Not exists.